09/09/15 **EXR**

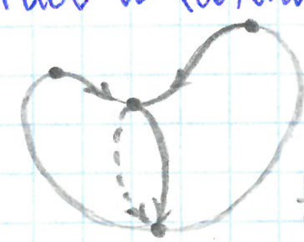
- $\dim M \neq 0$. If $\omega = d\theta$ for some $\theta \in \Omega^1$, then M must be non-comp. (or have bdry).
  An M admitting such a θ is called <u>exact</u>.
- Let $H: M \to \mathbb{R}$ be a C^∞ fn,
  X_H be the dual v.field to $dH \in \Omega^1(M)$
  $\rightsquigarrow$ let $\Phi^H: M \times \mathbb{R} \to M$ denote the corr. flow (called hamiltonian isotopies)
  Show that if $L \subset M$ is Lagrangian, then so is
  $L \times \mathbb{R} \hookrightarrow (M \times T^*\mathbb{R}, \omega_M \oplus \omega_{T^*\mathbb{R}} = dp \wedge dq$
  $(x,t) \longmapsto (\Phi^H(x,t), t, -H(\Phi^H(x,t)))$
- Show that H is constant along the flow of X_H.

<u>Last time</u>: Broad overview of Morse thry.

Given compact X, $f: X \to \mathbb{R}$, metric g on X, with f generic enough, one can construct a (co)chain ex.:

$\{\text{generators}\} \leftrightarrow \{\text{crit. pts}\}$

$d \longleftrightarrow \#$ of ∇ flows



s.t. $H_\bullet^{Morse}(X, f, g) \cong H_\bullet(X, \mathbb{Z})$

<u>Today</u>: We'll again fantasize about 1-dim'l moduli spaces of holomorphic strips (polygons)

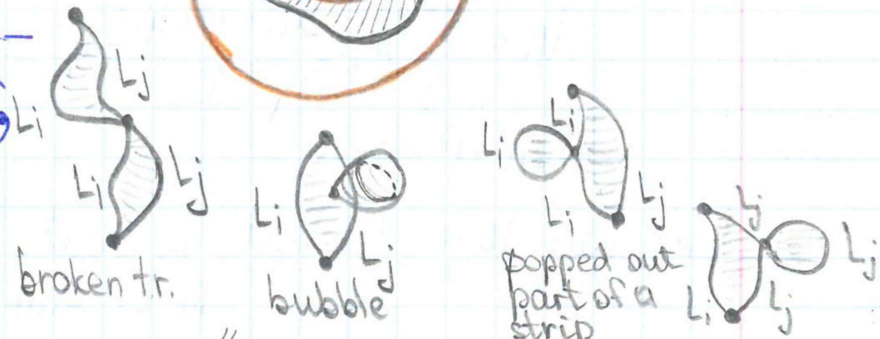
<u>Recall</u>: Fix $L_1, \ldots, L_n \subset M$, (M, ω, J), J compat. with ω
Fix i, j $\quad$ transverse Lagrangians

We studied $\left\{ u: \mathbb{R} \times [0,1] \to M \text{ s.t. } u(t,0) \subset L_i,\ u(t,1) \subset L_j,\ \lim_{t \to \pm\infty} u(t,\cdot) \subset L_i \cap L_j,\ du \circ i_c = J \circ du \right\} / \mathbb{R}$

Let's say that for some reason, we can pick out a 1-$\dim_{\mathbb{R}}$ space of such strips.

What does a compact'n of this 1-dim space look like? — We add "broken trajectories", "bubbles", "popped out parts"

(BUBBLES on edges are not supposed to appear)



broken tr.

bubble

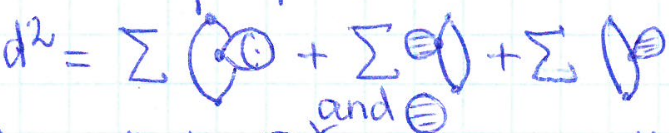
"energy concentrating at a point"

popped out part of a strip

"energy concentrating on an edge"

So it seems like doing Morse thry as usual (defining a d s.t.

$$dp = \sum_q \# \left\{ \begin{matrix} \text{holom. strips} \\ \text{from } q \text{ to } p \end{matrix} \right\} q) \text{ results in } d^2 \neq 0 \,!$$

Because $d^2 = \sum$ 

But if we a priori know that and cannot appear, then $d^2 = 0$.

That is the case for a so called <u>exact</u> mf with <u>exact</u> L_i.

<u>Def</u> M is <u>exact</u> if $\exists$ 1-form θ: $\omega = d\theta$.
$(M = (M, \omega))$

<u>Def</u> $L \subset M$ is called <u>exact</u> if $\exists f: L \to \mathbb{R}$: $\theta|_L = df$.

HARD MATH was when compactifying and classifying added "strips".

Thrm $\exists$ a setup (a perturbated version of what've been already told), in which if we define

$$CF^*(L_0, L_1) = \bigoplus_{p \in L_0 \cap L_1} \mathbb{Z}[|p|]$$

$$dp = \sum_q \#\left\{\begin{matrix}\text{holom strip}\\ p \rightsquigarrow q\end{matrix}\right\} q$$

then $d^2 = 0$.

(We can take M exact, L_i exact.)
or L_i spin