09/14/15 **EXR**

L5

- Fix $(\mathbb{R}^{2n}, \omega_{std})$
  Let $GrLag(\mathbb{R}^{2n}) := \{V \subset \mathbb{R}^{2n}$ linear s.t. V is Lagr.$\}$
  Show: $GrLag(\mathbb{R}^{2n}) \cong U(n)/O(n)$ and compute its π_1.

- Show that a dg cat w/ one obj is "the same thing" as a dg algebra (unital, assoc., not nec. gr. comm).

- Suppose $\exists \mu^3 : Hom(L_2, L_3) \otimes Hom(L_1, L_2) \otimes Hom(L_0, L_1) \to$
  $\to Hom(L_0, L_3)[-1]$

looks like a differential of μ^3

so, it is condition of "being associative up to hmtpy"

s.t. $\mu^1 \circ \mu^3 + \mu^3(\mu^1 -, -, -) + \mu^3(-, \mu^1 -, -) + \mu^3(-, -, \mu^1 -) =$
$= \mu^2(\mu^2(-,-), -) \pm \mu^2(-, \mu^2(-,-))$

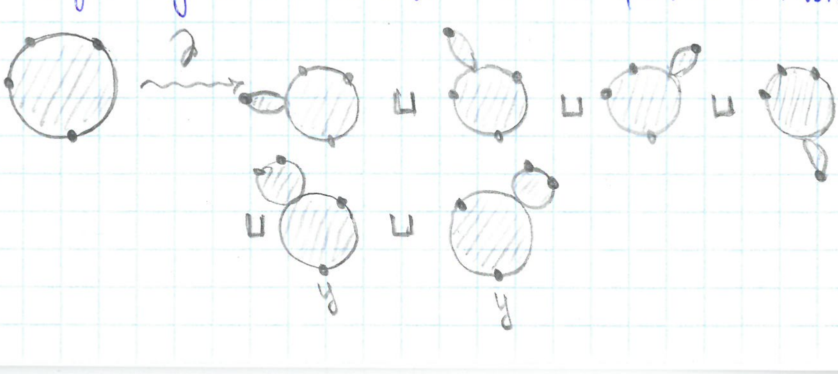
Show that $Hom_{H^*(\mathcal{C})}(X, Y) := H^* Hom_{\mathcal{C}}(X, Y)$ is a cat enriched over gr. abelian groups.
($\mathcal{C}$ is originally an A_∞ cat.)

Last time: we defined $\underset{\#\{\text{holo strips}\}}{\overset{}{\mu^1}} : Hom(L_i, L_j) \to Hom(L_i, L_j)[1]$

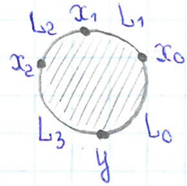
$\mu^2 : Hom(L_1, L_2) \otimes Hom(L_0, L_1) \to Hom(L_0, L_2)$

THIS DOES NOT DEFINE a DG Cat.

To prove $\mu^2(\mu^2, -) \pm \mu^2(-, \mu^2) = 0$ (assoc.) (turns out: NO associativity!)
might try to look at ∂ of 1-dim component of disks



That is, let $\mathcal{M}_{(x_0,x_1,x_2,y)} = \{u : D^2 \setminus \{4 \text{ pts on } \partial\} \to M\}$ s.t.



and $du \circ j = J \circ du$

$\mathcal{M}_{(x_0,x_1,x_2,y)}$ turns out to be a mf. Let $\mathcal{M}^1 \subset \mathcal{M}$ be the 1-dim components

$\exists$ compactification $\overline{\mathcal{M}^1}$ — 1-mf w/ bdry.

and $\partial\overline{\mathcal{M}^1}$ consists of what has been drawn on the prev. page.

Algebraically:

$x_i \in L_i \cap L_j$

$$0 = \mu^3(\mu^1 x_2, x_1, x_0) \pm \mu^3(x_2, \mu^1 x_1, x_0) \pm \mu^3(x_2, x_1, \mu^1 x_0) +$$
$$+ \mu^2(\mu^2(x_2, x_1), x_0) \pm \mu^2(x_2, \mu^2(x_1, x_0)) \pm \mu^1\mu^3(x_2, x_1, x_0)$$

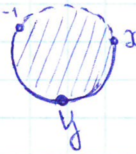
In particular, $\mu^2(\mu^2, -) \neq \mu^2(-, \mu^2)$ is general.

So, we see (by EXR) that μ^2 is only assoc. up to hmtpy. (given by μ^3).

But μ^3 is a choice. Now we'll show that composing ≥ 4 elts has coherent associativity.

<u>Def</u> Define $\mu^k : Hom(L_{k-1}, L_k) \otimes \ldots \otimes Hom(L_0, L_1) \to Hom(L_0, L_k)[2-k]$ of deg $2-k$

to be $\mu^k(x_{k-1}, \ldots, x_0) = \sum_{y \in L_0 \cap L_k} \# \Big\{$



$\Big| \; x_i \in L_i \cap L_{i+1}, \; y \in L_0 \cap L_k \Big\}$

<u>Thrm</u> (combinatorics after analysis)

These operations satisfy the following:

$$0 = \sum_{\substack{u+t+r=k \\ s=r+u+1}} \mu^s \circ (\mathbb{1}^{\otimes r} \otimes \mu^t \otimes \mathbb{1}^{\otimes u}) \qquad ☆$$

Def an A_∞ cat / k $\mathcal{C}$ is the data of:

- $\mathrm{Ob}\,\mathcal{C}$
- $\mathrm{Hom}_{\mathcal{C}}(L_0, L_1) =$ gr. k-mod
- $\forall$ $(k+1)$-tuples of objects $L_k, \dots, L_0 \in \mathrm{Ob}\,\mathcal{C}$:
  a linear map $\mu^k : \mathrm{Hom}(L_{k-1}, L_k) \otimes \dots \otimes \mathrm{Hom}(L_0, L_1) \to$
  $\to \mathrm{Hom}(L_0, L_k)[2-k]$
- condition $(\star)$