EXR • Consider the embedding $T^{n+1} \hookrightarrow S^{2n+1} \subset \mathbb{C}^{n+1}$ to the product of $(n+1)$ circles of radius $\frac{1}{\sqrt{n+1}}$

The quotient $T^{n+1}/S^1 \to S^{2n+1}/S^1 \to \mathbb{CP}^n$ realizes ~~the~~ an embedding $T^n \hookrightarrow \mathbb{CP}^n$

• Show this torus (called "Clifford torus") is Lagr (w/ sympl form induced by usual Kähler str're)

• Show that $[\underbrace{1:\ldots:1}_{i}:z:1:\ldots:1] = D_i$ w/ $|z| \le 1$ is a holo disc in $\mathbb{CP}^n$ w/ bdry on the Cliff torus

• $T\mathbb{CP}^n|_{D_i} \cong D_i \times \mathbb{C}^n$. and $T(\text{Cliff torus})|_{\partial D_i}$ gives a map $S^1 = \partial D_i \to GrLag(\mathbb{C}^n)$.
What elt of $\pi_1(GrLag(\mathbb{C}^n)) = \mathbb{Z}$ does this represent?

"—ARE THERE ANY QUESTIONS? FOR EXAMPLE, WHY ARE WE DOING THIS?
—OK, WHY ARE WE DOING THIS?
—FOR NO REASON. HAHA, I'M SUCH AN ASSHOLE"

<u>Last time</u> $\rightsquigarrow$ str're of an A_∞ cat (idea)

<u>TO DO</u>: Gradings: $CF^\bullet(L_0, L_1)$
i.e. if $p \in L_0 \pitchfork L_1$, $|p| = ?$

Transversality: Why is $\mathcal{M} = \{u : \underbrace{D \setminus \{z_0, \ldots, z_n\}}_{\cup \, \partial D} \to M\}$ a C^∞-mf?

Related: What if $L_0 \not\pitchfork L_1$?

Gluing: How do we define $\overline{\mathcal{M}}$?

Signs: How do we orient $\mathcal{M}$?

For now, assume that these issues can be resolved.

Def (INCORRECT, BUT THE IDEA)

Fix M exact, then the Fukaya category of M has objects $\mathrm{Ob}\,\mathcal{F}uk(M) = \{ L \overset{\text{exct}}{\subset} M \}$,

$\mathrm{Hom}^\bullet(L_0, L_1) = CF^\bullet(L_0, L_1)$

and A_∞ operations μ^k as defined last class.

So def from last time is of a "non-unital A_∞-cat".

Quick fix: Say an A_∞-cat $\mathcal{C}$ is unital if $H^\bullet\mathcal{C}$ is.

BABY FORM OF HOMOLOGICAL MIRROR SYMMETRY

Conj (Kontsevich '94 ICM)

det(T^*X) can be trivialized

$c_1(T^*X) = 0$

$\forall$ Calabi-Yau X $\exists$ Calabi-Yau $X^\vee$ and equivalences of A_∞-cats: $D^\pi \mathcal{F}uk(X) \cong D^b\,\mathrm{Coh}(X^\vee)$

$D^b\,\mathrm{Coh}(X) \cong D^\pi \mathcal{F}uk(X^\vee)$

means dg-enhancement of "classical" $D^b\,\mathrm{Coh}(X)$ (stable ∞-cat)

D^π is closure under cones & ...

1. Understanding $CF^*(L,L)$
(PSS isomm, pearl complex)
2. Symplectic homology, CY cats,
Abouzaid's generation criterion
(Hochschild (co)homology of a cat)
$(HH_\bullet(\mathcal{F}uk(M)) \cong SH^\bullet(M))$
3. Dynamics (Arnold's conj., existence of char. cycles)
4. HMS for toric Fano
(MATRIX FACTORIZATIONS)
"THAT'S PRETTY COOL"
5. Analytical details
(go through the TO DO list)
6. Arithmetic (H)MS + HMS for ell. curves
7. Nadler-Zaslow, Nadler
(more rep thry)
$\mathcal{F}uk^{inf}(T^*Q) \cong Shv^{const}(X) \subset D\text{-mod}(X)$
constructible
Q $\mathbb{R}$-analytic mf
"infinitesimal Fukaya cat"

LIST OF TOPICS