1. Understanding $CF^*(L,L)$: PSS isomorphism, pearl complex
(Analysis, Morse theory)

Example: If L, M exact, $HF^*(L,L) = H^*(L)$.

2. Symplectic homology, Calabi-Yau categories, Abouzaid's generation criterion
(Hochshild (co)homology of a category)
$$HH_\bullet(\mathcal{F}uk(M)) \simeq SH^*(M).$$

3. Dynamics. (Arnold's conjecture, existence of char. cycles)
4. HMS for toric Fano (Matrix factorization)
5. Analytical details
6. Arithmetic (H)MS + HMS for elliptic curves.
7. Nadler-Zaslow, Nadler
$$\mathcal{F}uk^{inf}(T^*Q) \simeq Shv^{const}(Q) \subset DMod(Q)$$
real anal.
category of constructible sheaves on X.

- email regarding PIAZZA
- fill out the ~~survey~~ TALK survey
- start posting solutions to PROBLEMS on PIAZZA

Lecture

TO-DO LIST:

- Smoothness (dimension)

How do we know $\mathcal{M} = \{u : D \to M,\ \bar\partial u = 0\}$ w/ boundary on Lagrang. etc. is a smooth manifold?

It's usually not.

Set-up: Fix unit disk $D \subset \mathbb{C}$.

Puncture $k+1$ ∂ points: $x_0, x_1, \dots, x_k \in \partial D$

$\hat{D} = D \setminus \{x_i\}$. Different choices of $\{x_i\}$ gives rise to non-equivalent mflds (in general):
$$D \setminus \{x_i\}_{i=0}^k \underset{\text{biholom.}}{\not\cong} D \setminus \{y_i\}_{i=0}^k$$

In general $\hat{D} \cong \hat{D}' \iff \exists$ an elmnt $f \in PSL(2,\mathbb{R})$ taking $\hat{D}$ to $\hat{D}'$.

Let $\tilde{R} := Conf_{k+1}(S^1) = \{(z_0 \dots z_k) \in S^1\}$ ordering

$\tilde{R} \circlearrowleft PSL(2,\mathbb{R}) = Aut(D)$

$R := \tilde{R}/PSL_2\mathbb{R}$ = moduli space of disks w/ $k+1$ marked pts on ∂

Rmk For $k+1 \geq 3$ this is not stacky, nor orbifoldy in any way. It's a mfld.

When $k+1 \geq 4$ we let:

$\mathcal{M} = \{(u, S)$ where S is a choice of a holom str. on $D \setminus \{k+1 \text{ pts}\}$ and u is a map $D \setminus \{k+1 \text{ pts}\} \to M$ satisfying J-hol, ∂ conditions etc. $\}$

$\xi \qquad \sim \xi_{(u,S)} = \{\Gamma_{C^\infty}(u^*TM)\}$

$\downarrow$

$\{(u,S),\ u: S \to M\ u\text{-}C^\infty\} \Rightarrow S \xrightarrow{u} M$

$\downarrow u^*TM \cong D \times \mathbb{R}^{2n}$

$S \in \mathcal{R} \quad \downarrow D \nearrow$

$\mathcal{F}_{(u,S)} = \mathrm{Hom}_{C^\infty}(TS, u^*TM) \ni du$

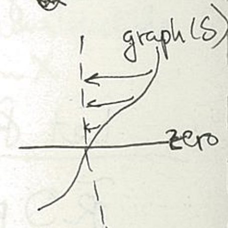
$$\begin{array}{ccc} \frac{1}{2}(du + J\circ du \circ j_S) & \in & \mathcal{F} \\ \uparrow u & \bar\partial & \downarrow \\ & & \{(u,S)\} \\ & & \downarrow \\ & & K \end{array}$$

So we have a vect bundle (∞-dim) $\mathcal{F}$ over a space (∞-dim) $\{(u,S)\}$ and a section $\bar\partial$.

We know $\{(u,S) \mid \bar\partial u = 0\}$

Should be a C^∞ mfld if graph$(\bar\partial) \pitchfork$ zero sect.

But it almost never is transverse.



(1) We'll perturb $\bar\partial$ eqn.

(2) We'll change the fibers of $\mathcal{F}$.

The way we address (1) is as follows.

We choose a 1-form on S w/ values in $C^\infty(M)$:

$Y \in \Omega^1_{dR}(S, C^\infty(M))$ a fnctn $f \in C^\infty(M)$ defines X_f via $df = \omega(\cdot, X_f)$.

The way we perturb is by now studying the diff eq.

$$\bar\partial(u) = X_Y + J_x \circ X_Y \circ j_S = X_Y + J_x \circ du \circ j_S$$

The way we change $\mathcal{F}$ is by looking at subbundle.

$\mathrm{Hom}^{0,1}_{C^\infty}(TS, u^*TM)$.

$$\begin{array}{ccc} v & \xrightarrow{Y} & f_v \in C^\infty(M) \\ \cap & \searrow^{X_Y} & \downarrow \\ TS & & X_{f_v} \in \Gamma(TM) \end{array}$$

J_x is a choice of almost complex str. on M $\forall x \in S$.

<u>Thm</u> $\exists$ (a lot of) choices of Y, J_x s.t. we obtain $\pitchfork$.

<u>Rmk</u> On $K+1=2$ case. The equation $\bar\partial u = 0$.

$u: \mathbb{R} \times [0,1] \longrightarrow M$

has a translation symmetry.

In this case we choose J_x, Y to be translation invariant.

i.e. to perturb the holomorphic strip $\mathbb{R} \times [0,1]$ equation we choose a time-dependent Hamiltonian

$Y = Y_s \quad s \in [0,1]$

and a time depend. almost $\mathbb{C}$ structure on M

$J_s, \ s \in [0,1]$.

<u>Rmk</u> · What is the dimension?

· How do we compactify?

Given $u: S \to M$ satisfy

$du + J_x \circ du \circ j_S = Y + J_x \circ Y \circ j_S$

Say we know $u \in$ 1-dim. component of $\{u$ satis.$\}$