Baris Abouzaid's generation criterion

<u>Thm</u> $\mathcal{B} \subset \mathcal{W} \subset$ Wrapped Fukaya cat.

$$HH_*(\mathcal{B},\mathcal{B}) \longrightarrow SH^*(M)$$

I'll define this map. $SH^*(M)$ has an identity coming from $H^*(M)$

If 1_{SH^*} is in the image, then $\mathcal{B}$ split generates $\mathcal{W}$.

This will follow from two ingredients.

(1) Cardy relation: Given $K \in \mathrm{ob}\,\mathcal{W}$, Δ natural coproduct

$$\begin{tikzcd} HH_\bullet^{*}(\mathcal{B},\mathcal{B}) \arrow[r, "H^*(\Delta)"] \arrow[d, "\text{open-closed}"'] & H^\cdot(\mathcal{Y}^r_K \otimes_{\mathcal{B}} \mathcal{Y}^l_K) \arrow[d, "H^\cdot(\mu)\ \text{composition}"] \\ SH^\circ(M) \arrow[r, "\text{Closed-open}"'] & HW^\circ(K,K) \end{tikzcd}$$

commutes.

(2) Algebraic lemma: If $H^\cdot(\mu)$ has $1_{HW^\cdot(K,K)}$ in image, then K is in split-closure of $\mathcal{B}$.

<u>Pf of Thm:</u> A fact: $1_{SH} \mapsto 1_{HW}$ under this map. (Closed-open.) Done. //

Geometric Setting:

(M,λ) Liouville manifold. ie, $\iota_Z \omega = \lambda$

$d\lambda = \omega$ and outside compact $K \subset M$,

Z defines flow st $M \setminus K$ is symplectization of ∂M.

ex $M = T^*L$

ex Affine variety.

So $M = M^{in} \bigcup_{\partial M} \partial M \times [1,\infty)$.

Lagrangians: $L = L^{in} \cup K \times [1,\infty)$

$K \subset \partial M$, $K \subset \partial M$

Legendrian Contact

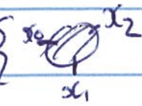
Hamiltonians: $H = r^2$, outside some compact set.

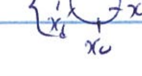
$M \supset \partial M \times [1,\infty) \ni (y,r)$.

$\mathcal{X}(L_0,L_1)$ time-one Ham. chords generate ham(L_0,L_1)

$CW^*(L_0,L_1)$ Wrapped Floer cochains

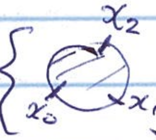
$HW^*(L_0,L_1)$ Wrapped Floer cohomology

Products: $\mu^2: x_0 \otimes x_1 \mapsto \sum \# \{$  $\}\, x_2$.

~~Coproducts~~ $\mu^d: x_{d-1} \otimes \cdots \otimes x_0 \mapsto \sum \# \{$  $\}\, x_d$

Same idea gives <u>coproduct</u> stucture!

$$\Delta : CF^{\circ}(L_1, L) \longrightarrow \bigoplus_{p+q=\bullet+\frac{1}{2}\dim M} CW^p(L_1, K) \otimes CW^q(K, L)$$

$$x_0 \longmapsto \sum \# \Big\{$$ 

$$\Big\} \, x_1 \otimes x_2.$$

K <u>fixed</u>

This is again a chain map.

It is also a <u>bimodule</u> map. What do we mean by bimodule? A bunch of coherent actions

$$\mathcal{B} \otimes \cdots \otimes \mathcal{B} \otimes Q \otimes \mathcal{B} \otimes \cdots \otimes \mathcal{B} \longrightarrow Q.$$

$\uparrow$ bimodule over $\mathcal{B}$

then a bimodule map $Q \longrightarrow P$ is a bunch of compatible maps

$$\mathcal{B} \otimes \cdots \otimes \mathcal{B} \otimes Q \otimes \mathcal{B} \otimes \cdots \otimes \mathcal{B} \longrightarrow P$$

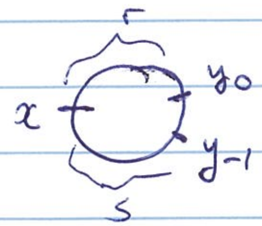
<u>Ex</u> Diagonal bimodule $\mathcal{B}$: $\mathcal{B}^{op} \otimes \mathcal{B} \to \mathrm{Chain}_k$

$$(L_1, L) \longmapsto \hom(L_1, L) =: \mathcal{B}(L_1, L).$$

<u>Ex</u> $\mathcal{Y}^K_r \otimes \mathcal{Y}^K_l : \mathcal{B}^{op} \otimes \mathcal{B} \longrightarrow \mathrm{Chain}_k$

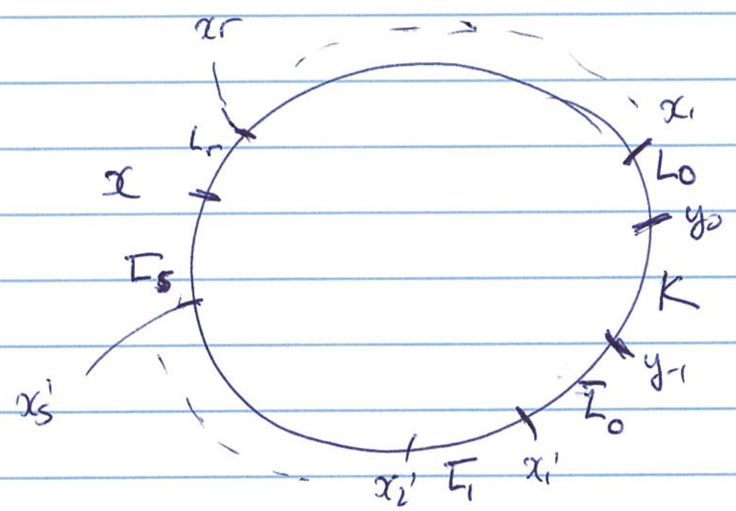
$$(L_1, L) \longmapsto CW(L_1, K) \otimes CW(K, L).$$

The higher maps of the bimodule map Δ are:

$$x_1'\otimes\cdots\otimes x_s'\otimes x\otimes x_r\otimes\cdots\otimes x_1 \longmapsto \sum \Big\{$$  $$\Big\}\, y_0\otimes y_{-1}$$

$$\underbrace{B\otimes\cdots\otimes B}_{s}\otimes Q\otimes\underbrace{B\otimes\cdots\otimes B}_{r}\longrightarrow P$$

where boundary conditions are



this set will be denoted $\mathcal{R}_{r|1|s}(\vec{x},\vec{y})$.

Symplectic Cohomology:

$$M \rightsquigarrow SH^\circ(M)$$

"Defn" $SH^\circ(M) = HW^\circ(\Delta_M, \Delta_M)$

where $\Delta_M \subset M \times \bar{M}$,

We think of this Δ_M as the identity of M in some category of symplectic manifolds.

We think of $SH^\circ(M)$ as $HH^\circ(\mathcal{F}uk)$, proven by Ganatra in some cases.

This is the open version of the conjecture $QH^\circ \cong HH^\circ$.

Ex $SH(\mathbb{C}^n) = 0$

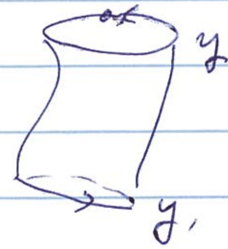
$SH(T^*X) \cong H_{n-*}(\Omega X)$

$SH(M_1 \times M_2) = SH(M_1) \otimes SH(M_2)$

Viterbo functoriality: Liouville cobordisms induce maps on SH°.

Another definition (used in Abouzaid's paper) $H_{S^1}(t,m) = H(m) + \underbrace{F(t,m)}_{F : S^1 \times M \to \mathbb{R} \text{ fixed}}$ to break S^1 symmetry.

Chain complex generated by time 1 periodic orbits of X_{H_S}.

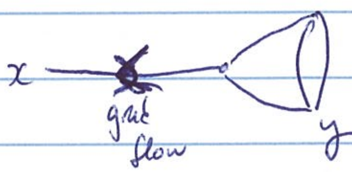


$$d(y_1) = \#\left\{ {}^{y_0}_{y_1} \right\} y_0.$$

maps $S^1 \times \mathbb{R} \longrightarrow M$ satisfying some PDE w/ asymptotic conditions

<u>Lemma</u> $\dim \mathcal{M}/\mathbb{R} = \deg(y_0) - \deg(y_1) - 1.$

<u>Rmk</u> Map $H^* \longrightarrow SH^*$ is PSS type.

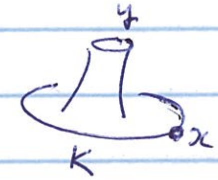


$$x \longmapsto \sum \#\left\{ {}_{x} \quad {}_{y} \right\} y$$

$$1 \longmapsto \sum \#\left\{ \begin{matrix}\text{rigid}\\ \text{holom}\\ \text{planes}\end{matrix} \quad {}_{y} \right\} y.$$

Closed-Open maps:

$$CO: SC^\bullet \longrightarrow CW^\bullet(K,K)$$

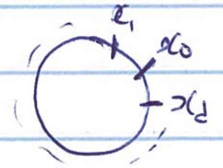
$y \longmapsto \sum \# \{$  $\} \, x$

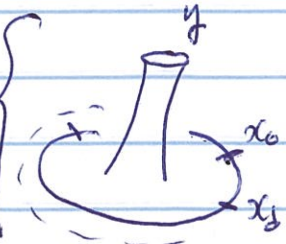
Chain map, since

 $+$  $= 0.$

Open-Closed map:

$$CC_*(B,B) \longrightarrow SC^\bullet(M)$$



$\longmapsto \sum \# \{$  $\} \, y$