Bars, II.

$$\begin{tikzcd} CC_\bullet(B,B) \arrow[r, "\Delta"] \arrow[d, "OC"] \arrow[dr, phantom, "\circlearrowleft"] & \mathcal{Y}^r_K \otimes_B \mathcal{Y}^l_K \arrow[d, "\mu"] \\ SH^\bullet(M) \arrow[r, "CO"] & CW^\bullet(K,K) \end{tikzcd}$$

Some comments:

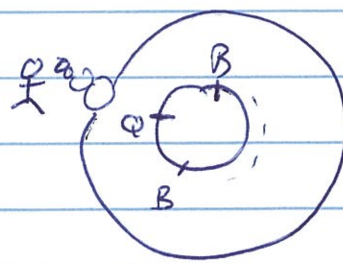
$$\mathcal{Y}^r_K \otimes_B \mathcal{Y}^l_K = R \otimes_B L \text{ "}= R \otimes B \otimes \cdots \otimes B \otimes L\text{"}$$

$$= \bigoplus R(L_d) \otimes B(L_{d-1}, L_d) \otimes \cdots \otimes B(L_0, L_1) \otimes L(L$$

this is a chain complex, differential given by module actions and B's μ^k.

The bimodule $CC_*(B, Q)$ = Hochschild chains of B w/ coeff in Q.

$$= \bigoplus Q \otimes B \otimes \cdots \otimes B$$



$\deg(q \otimes a_0 \otimes \cdots \otimes a_e) = \deg(q) + \sum \|a_i\|$.

where $\|a\|$ $\deg(a) + 1$

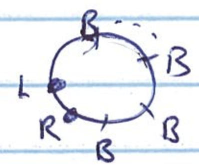
Q is a bimodule, i.e., an A_∞ functor

$$B^{op} \otimes B \longrightarrow Chain_K$$

It has structure maps $\cdots \otimes B(x, x_1) \otimes Q(x, x') \otimes B(x', x_1') \otimes \cdots \longrightarrow Q(x_s, x_r')$

What is degree of Δ? Comes from bimodule map

$$B \xrightarrow{\Delta} \mathcal{Y}^r_K \otimes_B \mathcal{Y}^l_K .$$

$CC(B, L \otimes R) =$ 

main point $\longrightarrow$ S^1

$R \otimes_B L$

CC_* is functorial in second input, so we get a map

$$CC(B,B) \longrightarrow CC(B, \mathcal{Y}^r_K \otimes \mathcal{Y}^l_K) \simeq \mathcal{Y}^r_K \otimes_B \mathcal{Y}^l_K .$$

And Δ is of degree n. (Boris won't tell us why).

And the map

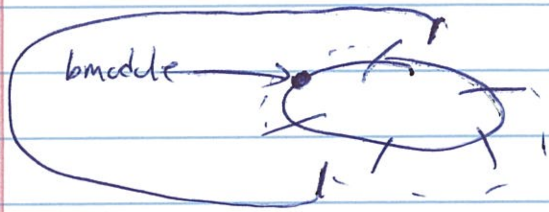
$$\begin{array}{ccc} CC_*(B,B) & \qquad & CW(L_{d-1}, L_0) \otimes \cdots \otimes CW(L_0, L_1) \\ \downarrow OC & & \downarrow \; n-d+1 \text{ degree} \\ SC^*(M) & & SC^* \end{array}$$

Thus the map $CC_*(B,B) \longrightarrow SC^*(M)$ has degree n.

Meanwhile, $SC^*(M) \xrightarrow{CO} CW^*(K,K)$ has degree 0.

(These degree comments are just assertions, speaker <u>not</u> proving them.)

Why does this diagram commute up to homotopy?

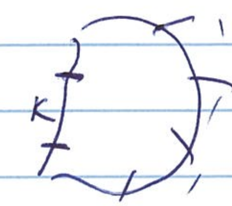


eat some of these elements $\rightsquigarrow$

$B \to \mathcal{Y}^\ell \otimes \mathcal{Y}^r$

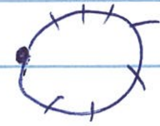
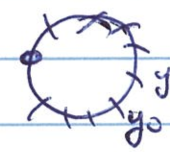
$CW(L', L) \to CW(L', K) \otimes CW(K, L)$

$\bullet \mapsto \sum \# \{$  $\}\ a \otimes b$

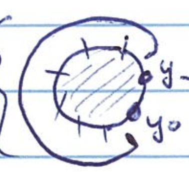


for higher maps $B \to \mathcal{Y}^\ell \otimes \mathcal{Y}^r$,

$B \otimes \cdots \otimes CW(L', L) \otimes \cancel{B} \cdots \otimes B \to \cancel{B \otimes} \otimes CW(L_1, K) \otimes CW(L_0, K) \cancel{\otimes \cdots \otimes B}$

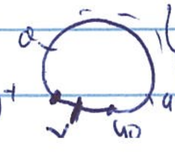
 $\longmapsto \sum \# \{$  $\}\ y_0 \otimes y_{-1}$

Then Δ sends

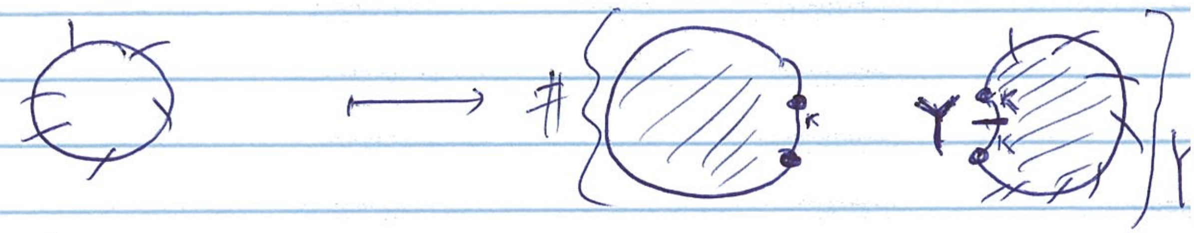
 to $\# \{$  $\}\ a \otimes \cdots \otimes a \otimes y_{-1} \otimes y_0 \otimes \cdots \otimes a$

leftover terms a

now gives $a \otimes \cdots \otimes a \otimes y_{-1} \otimes y_0 \otimes a \cdots \otimes a$, compose to count disks

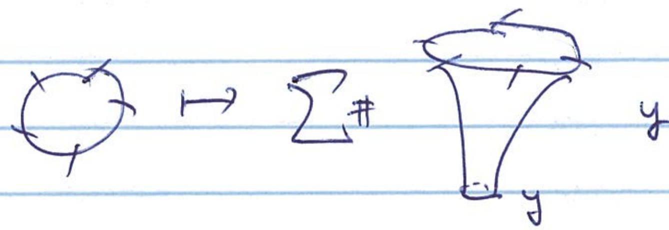
$\sum_Y \# \{$  $\}\ Y = \mu(a \cdots y_{-1} \otimes y_0 \cdots a)$

Baris summarizes this as taking

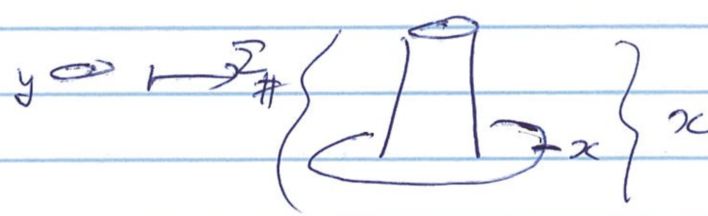


no disk, just
pts on S^1

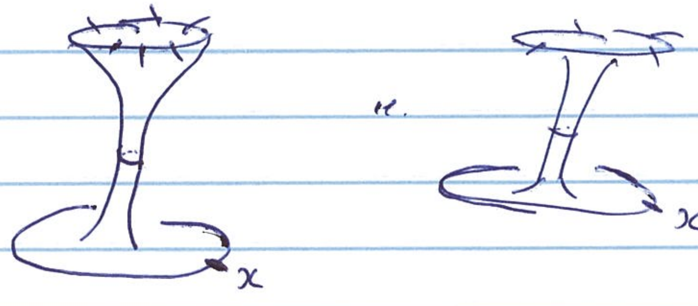
Meanwhile, OC:



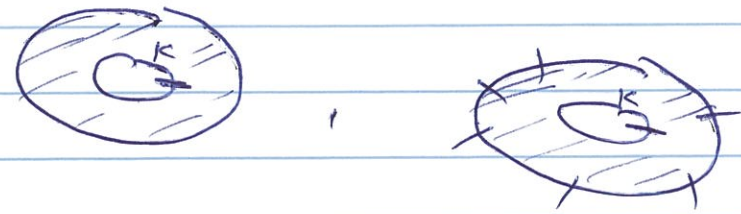
CO:



CO∘OC =

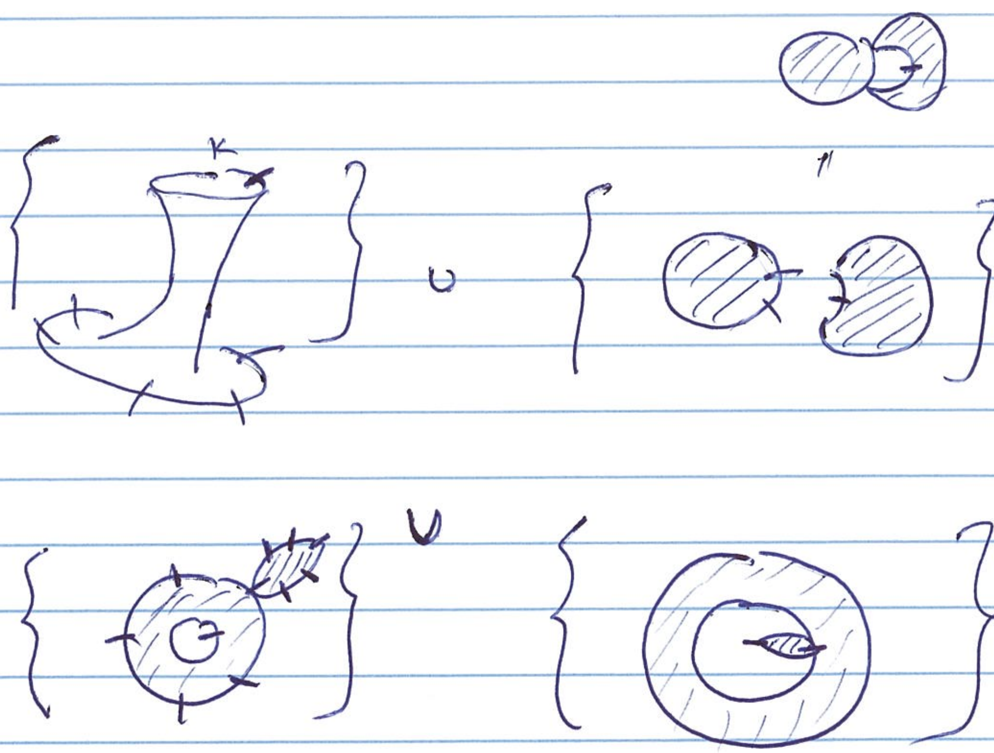


I'll draw these as ∂ of another moduli space.



What is ∂  ?

$\parallel$



d of CC.

d of CW'(K, K)

Abouzaid does this in two steps.
He has a big moduli of

w/ additional parameter t.

Then he passes to moduli of annuli.