

Fall 2015, Harvard University

**Math 277: Fukaya categories, sheaves,
and cosheaves (36 lectures)**

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This is a very preliminary and ambitious syllabus for the course. The original description of the course states: “After setting up the foundations for defining Fukaya categories, we will explore results showing that various Fukaya categories ‘glue.’ Little analytic background will be assumed, but we will attempt to cover the foundations.” If you replace the word “results” with “conjectures,” the description of the course is far more faithful.

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