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MATH 277 TALK TOPICS

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I've written below a few possible topics around which groups of you can collaborate. What I envision is that some of these topics will interest you, and that you will not be the only one. My hope, then, is that those of you with intersecting interests can speak with each other.

Within each group topic below, there are several ideas you can feel free to give a talk about. I want you to give a talk on whatever you think is most interesting to you, and beneficial to your education. So if your talk is purely algebraic (or purely dynamical, or purely geometric), that's totally fine, so long as you fit it into the big picture of mirror symmetry at some point.

You will learn much more material, I hope, then can fit into a one-hour talk. Do not pitch your talk to impressing me; pitch it so that your classmates will learn something. Keep in mind that a single expert audience member does not impart upon the whole audience expertise.

Contents

Date: Fall 2015, Harvard University.

1. Understanding Floer (co)homology and quantum cohomology: PSS and Pearl complex

Since the Floer complex is the morphism complex in the Fukaya category, a basic problem is to compute it. Or, even, to give it various interpretations.

Floer proved that, if ω restricted to $\pi_2(M, L)$ is zero, then the the Floer cohomology is equivalent to the usual singular cohomology of L (as a graded abelian group). Be mindful that the *cohomology* of homs in a dg category (or A_∞ category) is only part of the information, and that there is more information contained in the Floer cochains themselves when one accounts for the composition laws. Albers also proves this using techniques borrowed from Piunikhin-Salamon-Schwarz (PSS), so this isomorphism is sometimes called the PSS isomorphism. You could give a talk about this construction.

If you're hungry for examples, you can check out Cho's paper on computing Floer cohomology of the Clifford torus, which explains things well and shows how different spin structures give rise to different Floer cohomologies.

Also, an invariant of a symplectic manifold of interest is simply its quantum cohomology—this is a ring structure on cohomology of the manifold deformed by counts of holomorphic spheres.

References.

(1) Peter Albers, A Lagrangian Piunikhin-Salamon-Schwarz morphism and two comparison homomorphisms in Floer homology. `http://arxiv.org/abs/math/0512037`

(2) Jelena Katic , Darko Milinkovic, Piunikhin-Salamon-Schwarz isomorphisms for Lagrangian intersections. `http://www.sciencedirect.com.ezp-prod1.hul.harvard.edu/science/article/pii/S0926224504000725`

(3) For a brief and readable account: Francois Charette, Gromov width and uniruling for orientable Lagrangian surfaces `http://arxiv.org/pdf/1401.1953.pdf`

(4) The original pearl complex paper: Paul Biran, Octav Cornea, Quantum Structures for Lagrangian Submanifolds. `http://arxiv.org/abs/0708.4221v1`

(5) Cheol-Hyun Cho, Holomorphic Discs, Spin Structures, and Floer Cohomology of the Clifford Torus. `http://imrn.oxfordjournals.org.ezp-prod1.hul.harvard.edu/content/2004/35/1803`

2. Symplectic homology, Calabi-Yau categories, and Abouzaid's generation criterion

Rather than just look at moduli spaces of holomorphic polygons, you can examine holomorphic polygons with a puncture in the interior that eventually bounds a closed loop. This gives a relationship between the Hochschild homology of a Fukaya category and the symplectic homology of the symplectic manifold.

There are many topics you can touch on regarding this relationship. You can talk about Calabi-Yau categories in general. (References are Costello, and Ganatra's thesis.) Calabi-Yau categories are the kinds of categories that can guarantee that your category defines a two-dimensional topological field theory. They are "dualizable" in the sense of Lurie's formulation of the Baez-Dolan cobordism hypothesis (whose proof has been announced by Ayala-Francis). You can talk about why a Calabi-Yau category gives rise to a 2-D TFT.

You can talk about Abouzaid's generation criterion, which to date seems to be the most effective way to determine whether or not a collection of objects in the Fukaya category split-generates the wrapped Fukaya category.

Even if you don't get that far, you can just explain what Hochschild homology and cohomology are, and what they have to do with the open-closed and closed-open maps in wrapped Fukaya categories.

And, if you like, you can just talk about symplectic homology, which is an invariant for suitably convex symplectic manifolds.

Finally, you can also just choose to talk about what Hochschild cohomology has to do with deformations of categories.

References.

(1) Kevin Costello, Topological conformal field theories and Calabi-Yau categories. `http://www.math.northwestern.edu/~costello/0412149.pdf`
(2) M. Abouzaid, A geometric criterion for generating the Fukaya category `http://link.springer.com/article/10.1007%2Fs10240-010-0028-5`
(3) Sheel Ganatra, Symplectic cohomology and duality for the wrapped Fukaya category. `http://arxiv.org/abs/1304.7312`
(4) There are notes from a workshop on symplectic field theory here: `http://math.mit.edu/conferences/geometryworkshop/oldplan.html`

3. Dynamics in symplectic geometry

This is such a rich topic that you can take this any direction that you want.

You can talk about the motivation behind Arnold's conjectures, which says that the number of fixed points of a Hamiltonian system is way stronger than the number of fixed points given by the Lefschetz fixed point theorem—the sum of the Betti numbers, not their alternating sum. There are by now many proofs of this in many cases, and while the Floer theory proof seems most appropriate for this class, you can choose to present a different proof if you feel inclined.

Another proof you can give is the existence of closed characteristics in symplectic hypersurfaces. That is, if you have a proper Hamiltonian on $\mathbb{R}^{2n}$ with a regular value, the submanifold given by the level set of the regular value has a nowhere vanishing vector field on it given by the Hamiltonian flow. Does it have a periodic orbit? This is true if the hypersurface is of contact type, and can be shown to be true in many easier cases. You can feel free to talk about any case of appropriate difficulty.

References.

(1) Hofer and Zehnder, Symplectic Invariants and Hamiltonian Dynamics. . `http://link.springer.com.ezp-prod1.hul.harvard.edu/book/10.1007%2F978-3-0348-0104-1`
(2) So many others, but the book above does a very good job of referencing results.

4. Mirror symmetry for toric Fano varieties; matrix factorizations

This is probably the most well-understood version of mirror symmetry. You can very clearly see the presence of super-potentials, you can compute the equivalences of quantum cohomology and the Jacobi ring in specific examples, and you can learn toric geometry or things about matrix factorizations along the way. Talking about any of these aspects would be great.

Matrix factorizations are the algebraic mirror to symplectic manifolds with curved A_∞ structures. Toby Dyckerhoff has a very nice paper in which he identifies generating objects for nice matrix factorization categories. You can try to give a talk about that, or you can also try to give a talk about Orlov's result that matrix factorization categories are simply categories of singularities—they encode the geometry of a singular variety at its singularities.

As we'll see, curved A_∞ categories are not A_∞ categories, but they define an A_∞ category for every choice of a complex number. This is similar to a matrix factorization category, where a superpotential W defines a category for every possible value of W. The heuristic is that the mirror to W should be something like the moduli space of special Lagrangians with flat $U(1)$ connection, and the function W is a function encoding the μ^0 terms—i.e., measuring the volume of disks with boundary on a Lagrangian.

While this story is beautiful (and being actively developed), you can feel free to concentrate in these talks on purely algebraic results. If you can explain (via example) why coherent sheaves and perfect sheaves are different, great.

You can also give a talk about quantum cohomology of symplectic manifolds, and verify that Hochschild cohomology of a particular matrix factorization category agrees with quantum cohomology of the mirror.

References.

(1) T. Dyckerhoff, Compact generators in categories of matrix factorizations. `http://projecteuclid.org/euclid.dmj/1312481490`
(2) D. Orlov, Derived Categories of Coherent Sheaves and Triangulated Categories of Singularities. `http://www.math.nyu.edu/~tschinke/.manin/submitted/orlov.pdf`
(3) Kwokwai Chan, Naichung Conan Leung, Mirror symmetry for toric Fano manifolds via SYZ transformations `http://www.math.cuhk.edu.hk/~kwchan/SYZFano.pdf`

5. Analytical details

For the analysis-lover, you can feel free to give a talk on a rigorous proof of Gromov compactness in whatever setting you want—wrapped Floer theory, or symplectic homology, or Floer theory in the monotone setting, or even in the exact setting with compact Lagrangians.

You can also talk about Maslov indices and spectral flow. (To justify gradings on the Floer complex.)

You can also feel free to talk about regularity achieved via perturbing the Cauchy-Riemann equation.

References.

(1) Paul Seidel, Fukaya Categories and Picard-Lefschetz Theory.`http://www.ems-ph.org/books/book.php?proj_nr=12&srch=browse_authors|Seidel%2C+Paul`
(2) Michele Audin, Mihai Damian, Morse Theory and Floer Homology. `http://link.springer.com.ezp-prod1.hul.harvard.edu/book/10.1007%2F978-1-4471-5496-9`

6. Arithmetic mirror symmetry. Throw-in: Mirror symmetry for the elliptic curve.

Some of us are interested in doing mirror symmetry over spectra. If you can do such a thing, you want it to be interesting. A good testing case is to do mirror symmetry over $\mathbb{Z}$ and see if you can detect arithmetic.

I don't know much about this. But there are a few references.

Another thing that I'll throw into this group is mirror symmetry for the elliptic curve.

References.

(1) Y. Lekili, T. Perutz, Arithmetic mirror symmetry for the 2-torus.`http://www.ma.utexas.edu/users/perutz/HMSAug5.pdf`

(2) D. Wan, Arithmetic Mirror Symmetry. `http://www.math.uci.edu/~dwan/borel.pdf`

7. Nadler-Zaslow and Nadler

The main result is that, on analytic manifolds Q, there is an equivalence of A_∞ categories between

(1) The dg category of constructible sheaves on Q with respect to subanalytic stratifications, and
(2) A Fukaya category of T^*Q called the "infinitesimally wrapped" Fukaya category.

This should consist of two or three talks, and lends itself to multiple speakers. Do not concern yourself with too much of the analytical details, as some foundations are still being laid.

One of the motivations of this result is to test the hypothesis that the (appropriately dg-enhanced) category of nice D-modules on a space should be like the Fukaya category of the cotangent bundle of that space. (Both are thought of as "quantizations" of the usual categories of sheaves.) The connection to D-modules is that regular holonomic D modules are equivalent to constructible sheaves.

A nice application of this theorem is that any compact exact Lagrangian in T^*Q is equivalent to the zero section Q in the Fukaya category. Depending on your interests, you can choose to simply talk about this corollary and its proof.

Finally, there is a large philosophical difference between this infinitesimally wrapped category and other Fukaya categories that have non-compact Lagrangians. In this category, one nudges a Lagrangian ever-so-slightly near infinity. This is quite delicate. Moreover, one can also consider subcategories of objects that are asymptotic to some fixed $\Lambda \subset T^*Q$ near infinity.

References.

(1) `http://arxiv.org/abs/math/0612399v4`, Microlocal branes are constructible sheaves, David Nadler.
(2) `http://arxiv.org/abs/math/0604379`, Constructible Sheaves and the Fukaya Category, David Nadler, Eric Zaslow.
(3) As a warm up, you might want to consider Katsurirangan-Oh's paper: `http://link.springer.com.ezp-prod1.hul.harvard.edu/article/10.1007/PL00004822`, Floer homology of open subsets and a relative version of Arnold's conjecture.
(4) There was an MIT RTG run on this topic; there are notes available here: `http://math.mit.edu/conferences/geometryworkshop/2011/plan.html`