Introductory Meeting

Friday, September 16, 2016 2:02 PM

(Hiro Tanaka speaking)

Basics of Floer/Gauge Theory:

Study (space of) solns to some PDE

e.g: Fix (M, J), $u \in C^\infty(\Sigma, M)$

Does u satisfy $\bar\partial_J u = 0$?

Study

$\text{graph}(\bar\partial_J) \cancel{\pitchfork}$ zero section

in general, $\bar\partial_J$ not generic enough

One fix: Perturb $\bar\partial_J$

⇝ Leads to nastiness

(• invariance)

• lose natural functorialities

↳ eg: harder to prove Künneth

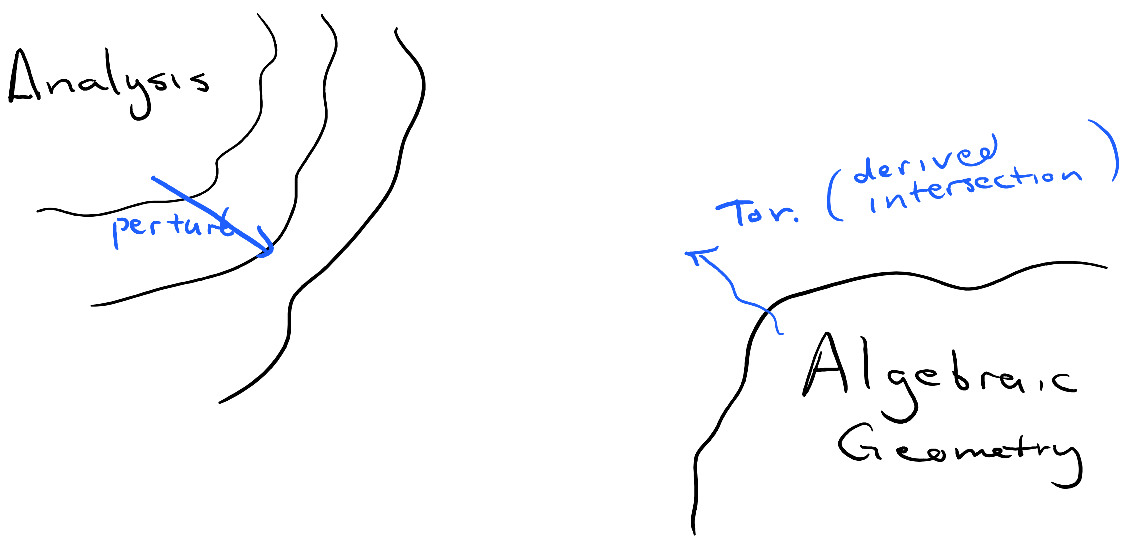
↳ e.g In Morse theory

$f_1 : M_1 \to \mathbb{R}$ $\qquad$ $f_1 + f_2 : M_1 \times M_2 \to \mathbb{R}$

$f_2 : M_2 \to \mathbb{R}$

(Exercise: If $g_1 \oplus g_2$ generic for $f_1 + f_2$, then Kunneth)

Another fix: Don't perturb, derive



$\mathbb{A}^1_{\mathbb{C}} = \mathbb{C} \ni p, q$

generically intersect 0 times

But $p = q$?

How about for

$\mathbb{A}^1_{\mathbb{F}} =$ ______ ?

t_q $\qquad$ $\mathrm{Spec}\,\mathbb{F}_q[t]$

Why might Tor come up?

If $X_i = \mathrm{Spec}(R_i)$ with

$$X_1 \longrightarrow X_3 \longleftarrow X_2$$

then

$$X_1 \underset{X_3}{\times} X_2 = \mathrm{Spec}(R_1 \underset{R_3}{\otimes} R_2)$$

<u>Notation</u>:

The chain complex computing

$$\mathrm{Tor}^{R_3}(R_1, R_2)$$

is often written

$$R_1 \overset{\mathbb{L}}{\underset{R_3}{\otimes}} R_2$$

Left derived

Back to $\mathbb{A}^1_{\mathbb{C}}$: $\chi(\mathrm{Tor}^{\mathbb{C}[t]}(\mathbb{C},\mathbb{C})) = 0$

Recreates intersection number

Goal: How do we bring "Tor" into Floer theory?

Turns out: To try to achieve goal, need homotopy theory!

(At least) two ways derived geometry has been developing:

(1) Pardon's work:

At end of day (or as a first step), we want to count the # of solutions (w/ signs) in the 0-dim'l component of sol'ns.

But counting is integrating.

If M_0 = 0-dim'l compact,

a fundamental class $[M_0] \in H_0(M)$

tells us how to count

$$H^\bullet(M, \mathcal{O}_M) \xrightarrow{\cap [M]} \mathbb{Z}$$

Generalize: If M not a mfld

$$H^\bullet(M; \mathcal{O}_M) \longrightarrow R \; ?$$

ground ring

Pardon gives framework for constructing

- $\mathcal{O}_M$
- $[M] \in H^\bullet(M; \mathcal{O}_M)^\vee = \hom(H^\bullet, R)$

(Even better

$[M] \in (C^\bullet(M, \mathcal{O}_M))^\vee$)

given the data of an implicit atlas on M.

Roughly, an implicit chart on M is

$$(\mathcal{U}, \varphi)$$

where • $\mathcal{U} \subset M$ open

• $\varphi : \mathcal{U} \xrightarrow{\sim} s^{-1}(0)$

where $s : M_J^{reg} \longrightarrow E_J$ is a C^∞ function from some C^∞-mfld to some Euclidean space

Hard parts:

• articulating compatibility of "transition maps"

Htpy theory — • constructing $[M]$

Analysis — • examples

(2) (⚠ Has not been shown to fit into Floer theory)

Spivak (Thesis), Lurie, Joyce, Carchedi–Roytenberg

Set-up

- What it means to be a derived mfld (inspired by alg. geom. "functor of pts") (Yoneda lemma)

- Rich category of derived mflds

Talk overview

(I) Basics of Floer Theory

II) Tor ; Serre's Intersection Formula

(Bonus: HKR Thm)

III) Ho(co)lims + Basics of htpcal ideas in chain complexes

IV) + V) Implicit atlases

→ analysis, explanation of examples

VI) K-Sheaves + Cech cohomology

VII) + VIII) Virtual Fundamental Cycles