

(Ian Banfield speaking)

## §1. Colimit

Defn: A colimit  $\varinjlim D$  for a functor

$D: \mathcal{J} \rightarrow \mathcal{C}$  is a universal arrow  
index category  
 from  $D$  to the diagonal functor

$\Delta: \mathcal{C} \rightarrow \mathcal{C}^{\mathcal{J}}$  — Function category  
Objects:  $F: \mathcal{J} \rightarrow \mathcal{C}$   
Morphisms: Nat. Transf

$\Delta_r =$  "constant diagonal map":  
 $\forall j, \Delta_r(j) = r$

i.e. a pair  $(\underset{\substack{\uparrow \\ \text{in } \mathcal{C}}}{r}, \underset{\substack{\uparrow \\ \text{in } \text{Mor}(\mathcal{C}^{\mathcal{J}}, \mathcal{C}^{\mathcal{J}})}}{u})$ ,  $u: D \rightarrow \Delta_r$   
" $\varinjlim D$ "

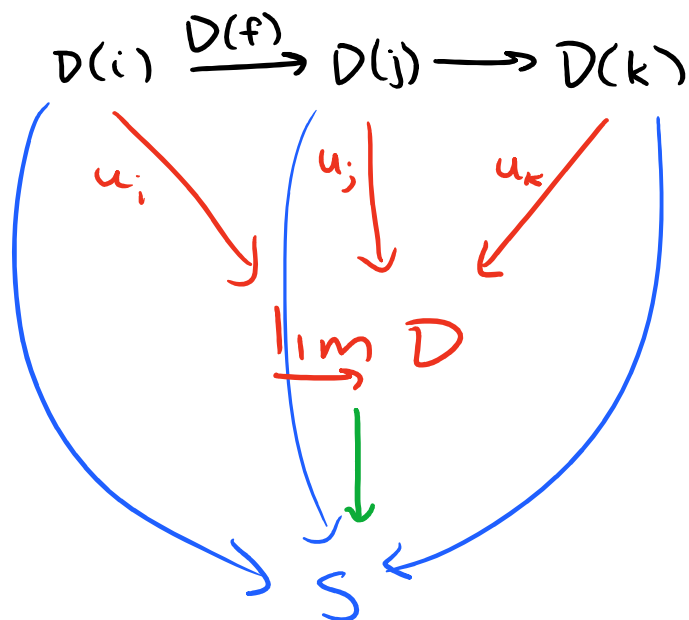
s.t.  $\forall$  morphisms  $D \xrightarrow{f} \Delta s$

$\exists$  a map  $f': r \rightarrow s$   
 such that this commutes

$$\left\{ \begin{array}{ccc} D & \xrightarrow{u} & \Delta r \\ \parallel & & \downarrow \Delta f' \\ D & \xrightarrow{f} & \Delta s \end{array} \right.$$



Q: What does this mean?



If  $S$  exists like this, then ! green arrow exists

e.g.  $\mathcal{C} = \text{Top}$   
 $\mathcal{J} = 1 \leftarrow 0 \rightarrow 2$

$$D = * \leftarrow S^n \hookrightarrow D^{n+1}$$

$\searrow \quad \downarrow \quad \swarrow$   
 $\lim D = S^{n+1}$

$\left( \begin{array}{l} \text{Think} \\ \lim D = * \sqcup S^n \sqcup D^{n+1} \end{array} \right. \left. \begin{array}{l} \text{---} \\ S^n \sim * \sim i(S^n \hookrightarrow D^{n+1}) \end{array} \right)$

e.g.  $D' = * \leftarrow S^n \longrightarrow *$

$\searrow \quad \downarrow \quad \swarrow$   
 $\lim D' = *$



$$\lim_{x \rightarrow 0} \frac{1}{x} = \infty$$

(Think  $* \sqcup S^n \sqcup *$  /  $* = S^n = *$ )

Given  $\gamma: D \rightarrow D'$  natural transf.

$$\text{for } i \in J, \quad \eta_i: \underset{\text{Top}}{\cap} D(i) \longrightarrow \underset{\text{Top}}{\cap} D'(i)$$

Fact: If we have a natural transf

$$\eta: D \rightarrow D'$$

s.t.  $\gamma_i$  are homeomorphisms

then  $\varinjlim D = \varinjlim D'$

In our example:

Have weak equivalence  $D \rightarrow D'$

BUT  $\varinjlim D \neq \varinjlim D'$

so this is httpy-theoretically BAD

- \* Motivates introduction of  $\text{htpy}$ -theoretic version