DGFT Lecture 4 Part 2 - Into to Implicit Atlases

Friday, October 21, 2016 2:04 PM

(Jake McNamara, ctd.)

Recall: Implicit Atlas for X = Hausdorff space

- A = Index set
- X_I = Hausdorff Space ⇝ thickening
- E_α = f.d. v.space ⇝ obstruction
- $s_\alpha : X_I \to E_\alpha$ ⇝ Kuranishi maps
- $U_{IJ} \subseteq X_I$ ⇝ footprints
- $\psi_{IJ} : (s_{J\setminus I}|_{X_J})^{-1}(0) \xrightarrow{\sim} U_{IJ}$ ⇝ footprint maps
- $X_I^{reg} \subseteq X_I$ ⇝ regular locus

satisfying

- Compatibility
- submersion

$$s_{J\setminus I} : X_J \longrightarrow E_{J\setminus I}$$

locally:

$$\mathbb{R}^{d + \dim E_I} \times \mathbb{R}^{\dim E_{J\setminus I}} \longrightarrow \mathbb{R}^{\dim E_{J-I}}$$

over $\psi_{IJ}^{-1}(X_I^{reg}) \subseteq X_J^{reg}$

- Covering

EXAMPLES:

① Smooth manifold X^n

Take $A = \{*\}$

$$X_* := X, \quad E_* = 0$$

$$s_* = 0, \quad \mathcal{U}_{\emptyset *} = X$$

and

n = virtual dimension

s_* locally $\mathbb{R}^n \longrightarrow 0$

①' Smooth mfld X^n with atlas $\{\mathcal{U}_\alpha\}_{\alpha \in A}$

Take

$$X_I = \bigcap_{\alpha \in I} \mathcal{U}_\alpha, \quad E_\alpha = 0$$

$$s_\alpha = 0, \quad \mathcal{U}_{IJ} = X_J \subset X_I$$

<u>Remark</u>: Say $X = *$, $A = *$

$$X_* = \mathbb{R}$$

$$E_* = \mathbb{R}$$

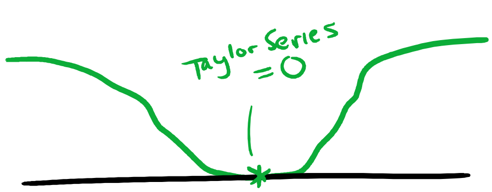
$$s_* : X_\alpha \longrightarrow E_\alpha$$

$$x \longmapsto e^{-1/x^2}$$

$$X_*^{reg} = \mathbb{R}$$

$$X_\emptyset^{reg} = \emptyset$$

$\Longrightarrow$ submersion axiom for s_* checked over $\emptyset$



Remembers <u>germ</u> not just ∞-jet

" X_I^{reg} = set of smooth points of X_I "

② Zero locus of section of v.bundle :

$$\begin{tikzcd} F \arrow[d] \\ Y \arrow[u, bend left, "\sigma"] \end{tikzcd}$$

smooth v.b.

$$X := \sigma^{-1}(0)$$

Choose $\tilde{F}$ s.t. $F \oplus \tilde{F}$ = trivial bundle $\begin{matrix} E\times Y \\ \downarrow \\ Y \end{matrix}$.

Take

$$X_\phi = X$$

$$X_* = \tilde{F}, \quad E_* = E, \quad s_* : \tilde{F} \to E$$

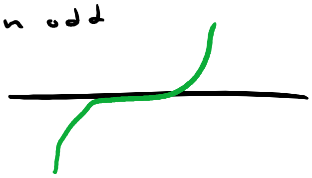
$$(y, \tilde{f}) \mapsto \sigma(y) + \tilde{f}$$

$$\left(\begin{matrix} X_\phi^{reg} = \text{where transverse} \\ X_*^{reg} = \text{all} \end{matrix} \right)$$

<u>Example</u> : $V(x^n)$ $\qquad X_\alpha = \mathbb{R}, \quad E_\alpha = \mathbb{R}, \quad s_\alpha(x) = x^n$

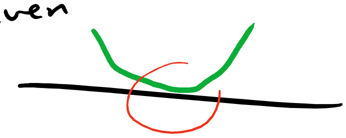
$$\operatorname{Spec}\left(\mathbb{R}[x]/x^n\right)$$

n odd

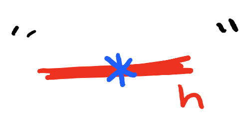


Expect: VFC = 1

n even



Expect VFC = 0



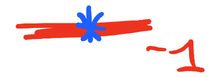
degree n thickening

<u>Example</u>: $\Omega_0 \mathbb{R}$, $\qquad X_\alpha = *, \quad E_\alpha = \mathbb{R}$

$$s_\alpha : * \mapsto 0$$

$$“\begin{tikzcd} \Omega_0\mathbb{R} \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & * \arrow[d] \\ * \arrow[r] & \mathbb{R} \end{tikzcd}”$$

$* \cap_{\mathbb{R}} *$

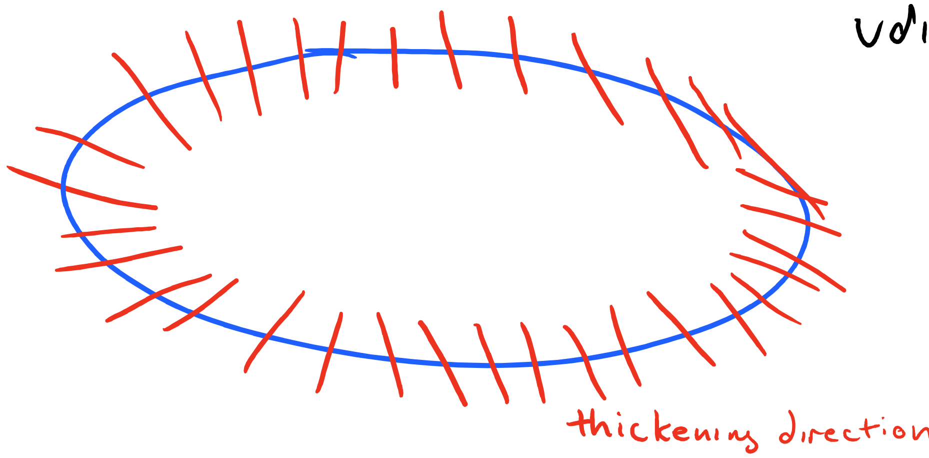


Generally, for $x_0 \in X$, can form $\Omega_{x_0} X$

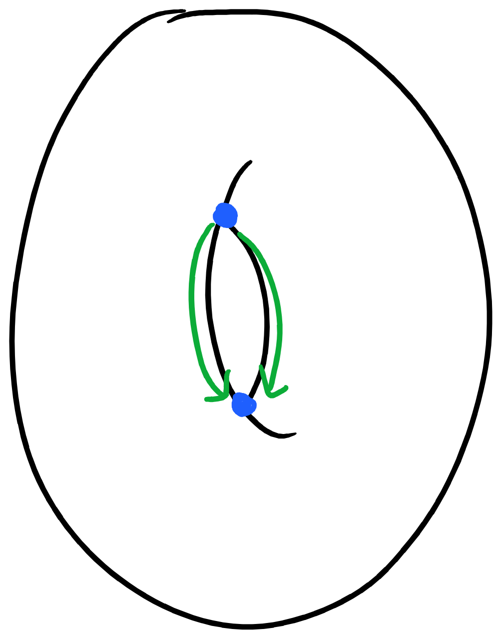
$X_\alpha = \{x_0\}$, $E_\alpha = T_{x_0}X$, $s_\alpha = 0$

Example: 0-locus of 0-section of Möbius

vdim = 0

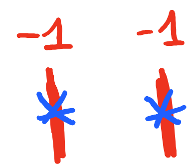


Picture for Morse homology



don't tilt

$X_\emptyset = * \quad *$ the two flow lines



Take $X_\alpha = * \quad *$

$E_\alpha = \mathbb{R}$

$s_\alpha = 0$

i.e. 1-parameter family of perturbations where flow lines only exists at one point

Remark: Only using 1-parameter family of perturbations. Needs analytic machinery to show this is OK. (Doesn't take into account all perturbations)