DGFT Seminar Outline

September 15, 2016

This is a seminar on derived geometry approaches to Floer theory. The seminar will consist mainly of background to Pardon's work, and a reading of Pardon's work. There are also optional topics we can cover at the end, though they would be on unwritten and perhaps speculative material. The talk schedule is subject to discussion and change, but I wrote something up for concreteness and direction. I can try to write up a list of resources and references later. – Hiro

Contents

1 Basic set-up of Floer theory; where transversality comes in

Explain various theories which should benefit from a new framework for transversality: Morse theory, Lagrangian Floer theory, contact homology. Where do transversality issues come in? How do we traditionally circumvent them? Explain basic Fredholm theory, an example of indices, and how one proves the existence of regular Fredholm sections in above settings. If possible, discuss with an eye toward talks 4 and 5. Note that different transverse perturbations result in cobordant (not diffeomorphic) intersections.

2 Tor; Serre's intersection formula

This is not a prerequisite for understanding Pardon's work, but will motivate why homotopical language and derived ideas enter the picture. Review how tensor product of rings is the same thing as intersection of affine varieties. Explain Serre's intersection formula. How does it circumvent the need to perturb an intersection into a transverse one? Examples illustrate some heuristic meaning to what higher Tor groups mean when both rings are concentrated in degree zero. Examples should include: A point intersecting itself, and the diagonal intersecting itself (a.k.a Hochschild homology of a commutative ring; use HKR Theorem to see the "differential forms" explanation, and if time allows, describe the circle action giving rise to the deRham differential).

3 Homotopy (co)limits, or why does homotopical language help

Pardon's work is very concretely rooted in cochain complexes, but not everybody is well-versed in the homotopy theory thereof. So we should review: The

Dold-Kan correspondence (i.e., how to think of a chain complex as a space). Homotopy (co)limits and examples (e.g., why mapping cones are a good idea). How to construct a total complex to compute a homotopy (co)limit in cochain complexes. (This will lead into how to compute global sections of H-sheaves.) This talk can also motivate the terminology "derived." Things are typically (and classically) derived to give rise to long exact sequences. But one can take the philosophy that all long exact sequences come from fiber sequences and homotopy groups, or from cofibration sequences and homology. Tor is a way to take pullback diagrams of algebraic spaces, while we'll see homotopy (co)limits to compute global sections of homotopy (co)sheaves in later talks.

4 Implicit atlases for manifolds, I.

A definition of implicit atlases (a la Pardon). First do it without the finite automorphism groups. Explain how Morse theory gives an example, even in the non-generic case. Be as detailed as possible.

5 Implicit atlases for manifolds, II.

A definition of implicit atlases, now with finite automorphism groups. Explain how J-holomorphic curves fit in, following Pardon's exposition. Be as detailed as possible, especially explaining some of the analysis that goes into proving the elements of being an implicit atlases.

6 H-sheaves and Cech cohomology of sheaves

Explain, from the basics, the Cech nerve associated to an open cover, and Cech cohomology. It's best to consult some classical tests that compare Cech cohomology to other cohomology theories, to shed some light on why we might be using it in derived geometry. Define H-sheaves, and what data is required to construct a global section of one.

7 Virtual Fundamental Cycles, I

Review Pardon's definition, and what data is required to give a VFC. Explain how to construct one in the simple setting of when your implicit atlases arises from a section of a single smooth vector bundle. Explain how to "integrate." If time allows, connect to what algebraic geometers usually call a VFC.

8 Virtual Fundamental Cycles, II

Now explain how to give a VFC in general, and—building on implicit atlases for the examples like Morse Theory and holomorphic curves—explain how the examples fit into this framework. This can take another talk if needed.

9 Optional topic: What's needed to set up a Kunneth formula

Unwritten material: What more data do we need to throw in to formally prove a Kunneth formula for Morse/Floer theory using only derived geometry, hence no perturbations.

10 Optional topic: Spivak/Lurie's approach to derived geometry

Explain how an implicit atlas on a manifold is the same thing as a derived manifold in the sense of Spivak. This requires explanation of C^∞ rings. This may require two talks.

11 Optional topic: How a VFC in Pardon's sense defines a derived cobordism class in the sense of Spivak