5 Due Tuesday, October 1

5.1 Multiple choice.

You get $+1$ for every response correctly chosen, and -1 for every response you incorrectly choose.

Based on what we have defined in this class, which of the following notions make sense?

(a) For a function between two metric spaces to be continuous.

(b) For a function between two topological spaces to be continuous.

(c) For a function between two metric spaces to be continuous.

(d) For a function between two topological spaces to be convergent.

(e) For a function between two topological spaces to be open.

(f) For an open subset of a topological space to be convergent.

(g) For a subset of a metric space to be open.

(h) For a subset of a topological space to be open.

(i) For a sequence in a metric space to be convergent.

(j) For a sequence between two metric spaces to be convergent.

(k) For a sequence between two metric spaces to be continuous.

(l) For a topological space to be convergent.

(m) For an element of a metric space to be convergent.

(n) For an element of a topological space to be continuous.

5.2 Multiple choice.

You get +1 for every response correctly chosen, and −1 for every response you incorrectly choose.

Let X be a set and $E \subset X \times X$ an equivalence relation. Which of the following is true?

(a) If $A, A' \subset X$ are two equivalence classes, then either $A \cap A' = \emptyset$ or $A = A'$.

(b) x and x' are in the same equivalence class if and only if $x \sim x'$.

(c) $X/\sim$ is empty if and only if X is empty.

(d) The function $X \to X/\sim$ is an injection.

(e) The function $X \to X/\sim$ is a surjection.

5.3 Proof (10 points)

Let $(X, \mathcal{T}_X)$ be a topological space. We say that $(X, \mathcal{T}_X)$ is *Hausdorff* if and only if the following holds:

For any $x, x' \in X$ with $x \neq x'$, there exist open sets $U, U' \subset X$ such that

1. $x \in U$ and $x' \in U'$, and
2. $U \cap U' = \emptyset$.

(a) Suppose that d_X is a metric on X, and $\mathcal{T}_X$ is the topology induced by d_X. (So $U \in \mathcal{T}_X$ if and only if U is a union of open balls.) Prove that $(X, \mathcal{T}_X)$ is Hausdorff. (That is, any metric space is Hausdorff.)

(b) Let $(X, d_X) = (\mathbb{R}, d_{std})$ and consider the following equivalence relation on $\mathbb{R}$: We say that $x \sim x'$ if and only if there is a *non-zero* real number t such that $tx = x'$. Show that $X/\sim$ is not Hausdorff.

5.4 Extra Credit (5 points)

Give an example of two topological spaces X and Y, together with a continuous surjection $f : X \to Y$, where Y does *not* have a quotient topology induced by any equivalence relation on X.

5.5 Extra Credit (5 points)

True or false: If X and Y are Hausdorff topological spaces, then their product is Hausdorff. You get no credit unless you justify your answer.

5.6 Extra Credit (5 points)

True or false: Every non-Hausdorff topological space is the quotient of a Hausdorff topological space.