6 Homework due Tuesday, October 8

Set-up: Let $\mathbb{R}^2 \setminus \{(0,0\}$ denote the set of points in $\mathbb{R}^2$ that are not the origin. Define an equivalence relation by declaring that

$$(x_1, x_2) \sim (x_1', x_2')$$

if and only if there exists a non-zero real number t such that

$$tx_1 = x_1' \qquad \text{and} \qquad tx_2 = x_2'.$$

We let

$$X := (\mathbb{R}^2 \setminus \{(0,0\})/\sim$$

denote the quotient space. (So X is given the quotient topology.)

On the other hand, define an equivalence relation $\sim_{S^1}$ on S^1 by declaring that

$$(x_1, x_2) \sim_{S^1} (x_1', x_2')$$

if and only if one of the following holds:

$$(x_1, x_2) = (x_1', x_2') \qquad \text{or} \qquad (x_1, x_2) = (-x_1', -x_2').$$

We let

$$Y := S^1 / \sim_{S^1} .$$

Problem: Exhibit a homeomorphism from X to Y.
(This verifies that the two topologies we have put on $\mathbb{R}P^1$ are equivalent.)