Homework 8: Due Tuesday, October 29

Writing assignment

Try writing/drawing examples of closed subsets of $\mathbb{R}^2$. What kind of intuition do they convey? What is the closure of a set? Why might thinking about closed sets instead of open sets be useful?

7.1 Proof (10 points)

A topological space is called *compact* if every open cover admits a finite subcover.

In other words, if $(X, \mathcal{T}_X)$ is a topological space, it is compact if and only if the following hold: For every open cover $\{U_\alpha\}_{\alpha \in \mathcal{A}}$ of X, there exists some finite subset $\{\alpha_1, \ldots, \alpha_n\} \subset \mathcal{A}$ so that $U_{\alpha_1} \bigcup \ldots \bigcup U_{\alpha_n}$ is an open cover of X.

Note that, given a topological space X, it makes to sense to ask whether a subset $A \subset X$ is compact—by giving A the subspace topology.

Problem: Let $f : X \to Y$ be continuous. Show that any compact subspace of X is sent to a compact subspace of Y.

7.2 Extra credit (1 point)

Give an example of a compact space.

7.3 Extra credit (1 point)

Give an example of a topological space and an open cover of that space.

7.4 Extra credit (1 point)

Give an example of a topological space and a closed subset of that space.

7.5 Extra credit (5 points)

Prove or disprove: If X and Y are compact, then so is the product $X \times Y$.