

Homework 9: Due Tuesday, November 5

Writing assignment

Choose a topic from this class that you feel you have not had time to digest properly. Think about it; try to understand why you feel you have not had time to digest this topic properly (whether that be for personal reasons or mathematical reasons). Now take the time to think about this topic and try to understand it better. Write about what you were able to explore and learn.

8.1 Proof (10 points)

Suppose X is compact and Y is Hausdorff.

1. Show that if $A \subset X$ is closed, then A is compact.
2. Show that if $B \subset Y$ is compact, then it is closed.
3. Let $f : X \rightarrow Y$ be a continuous bijection. Then show f is a homeomorphism (still under the assumption that X is compact and Y is Hausdorff). (Hint: Use the previous parts of this problem, and use last week's proof, too.)

8.2 Extra credit (1 point)

Give an example of a closed and bounded subset of \mathbb{R} .

8.3 Extra credit (1 point)

Give an example of a closed but not bounded subset of \mathbb{R} .

8.4 Extra credit (1 point)

Give an example of a bounded but not closed subset of \mathbb{R} .

8.5 Extra credit (1 point)

Give an example of a subset of \mathbb{R} that is neither closed nor bounded.

8.6 Extra credit (1 point)

Given an example of a subset of \mathbb{R} that is neither open nor closed.

8.7 Extra credit (5 points)

Let X and Y be topological spaces, and let $A \subset X$ and $B \subset Y$ be closed subsets. Prove or disprove: $A \times B \subset X \times Y$ is closed.

8.8 Extra credit (5 points)

Given an example of a (not open) cover $\{A_\alpha\}_{\alpha \in \mathcal{A}}$ of a space X such that the continuous bijection

$$\coprod_{\alpha} A_{\alpha} / \sim \rightarrow X$$

is not a homeomorphism.