Homework 11: Due Tuesday, November 19

Time recommendation: An hour a day.

Writing

Explore the ideas of connectedness and path-connectedness. Why might giving a name to, or identifying, these properties be useful? Does it seem natural to come up with these properties? What questions do you have about them? Can you answer these questions?

Multiple Choice

Is online.

Proof (10 points)

Let (X, d) be a metric space, and fix a sequence of elements

$$x_1, x_2, \ldots$$

in X. Choose also $x \in X$.

Recall that we say this sequence *converges to* x if, for every $\epsilon > 0$, there exists $N > 0$ so that

$$i > N \implies d(x_i, x) < \epsilon.$$

(a) Suppose X is a metric space and that $A \subset X$ is a closed subset. Then for any convergent sequence $x_1, x_2, \ldots$ converging to x, if $x_i \in A$ for all i, then $x \in A$.

(b) Suppose X is compact, and let $f : X \to \mathbb{R}$ be a continuous function. Show that f achieves a minimum and a maximum—that is, there is some $x_{\min} \in X$ such that for all $x \in X$, $f(x) \geq f(x_{\min})$. Likewise, there is some $x_{\max} \in X$ so that for all $x \in X$, $f(x) \leq f(x_{\max})$.

11.1 Extra credit (1 point)

Give an example of a path-connected space that is also compact.

11.2 Extra credit (1 point)

Give an example of a path-connected space that is not compact.

11.3 Extra credit (1 point)

Give an example of a compact space that is not connected.

11.4 Extra credit (1 point)

Give an example of a non-compact space that is not connected.

11.5 Extra credit (5 points)

Let $B \subset \mathbb{R}^2$ be the set of those elements (x_1, x_2) for which $x_1 > 0$ and $x_2 = \sin(1/x_1)$. What is the closure of B? Prove it.