## Homework 13: Due Tuesday, December 3

Time recommendation: An hour a day.

## Writing (10 points)

This is your final writing assignment. Either
(I) Look back on any idea you feel you have not had time to digestexplore and write about that idea, or
(II) Look back on your own experience in this class, especially as your "non-mathematical" life interfered, interacted, or intersected with your mathematical life. What went right and what went wrong?

## Multiple Choice

None this week.

## Proof? (10 points)

Draw a sequence of pictures illustrating (i) what shape/surface you obtain by gluing the edges of the octagon from the in-class exercises (entitled "what the...?"), and illustrating (ii) why you obtain that shape/surface.

There is more than one way to reason through this gluing, so have fun with it. If you are a bad drawer, you may want to spend extra time on these drawings. Good drawings will definitely get some extra points.

## Extra Credit (5 points)

Let $X$ be a set with exactly two elements. Call the elements $a$ and $b$.
Prove that $X$ admits exactly 4 topologies.
Prove that, up to homeomorphism, there are exactly three topological spaces with exactly two elements.

## Extra Credit (5 points)

Let $X$ be a Hausdorff topological space. Prove that any subset $\{x\} \subset X$ consisting of exactly one point is a closed subset of $X$.

