Homework 13: Due Tuesday, December 3

Time recommendation: An hour a day.

Writing (10 points)

This is your final writing assignment. Either

- (I) Look back on any idea you feel you have not had time to digest—explore and write about that idea, or
- (II) Look back on your own experience in this class, especially as your "non-mathematical" life interfered, interacted, or intersected with your mathematical life. What went right and what went wrong?

Multiple Choice

None this week.

Proof? (10 points)

Draw a sequence of pictures illustrating (i) what shape/surface you obtain by gluing the edges of the octagon from the in-class exercises (entitled "what the...?"), and illustrating (ii) why you obtain that shape/surface.

There is more than one way to reason through this gluing, so have fun with it. If you are a bad drawer, you may want to spend extra time on these drawings. Good drawings will definitely get some extra points.

Extra Credit (5 points)

Let X be a set with exactly two elements. Call the elements a and b.

Prove that X admits exactly 4 topologies.

Prove that, up to homeomorphism, there are exactly three topological spaces with exactly two elements.

Extra Credit (5 points)

Let X be a Hausdorff topological space. Prove that any subset $\{x\} \subset X$ consisting of exactly one point is a closed subset of X.