Writing assignment for Thursday, September 12

Homework for Thursday:

Consider the following definition:

Definition 3.7.1. Let (X, d) be a metric space, and fix a sequence of elements

$$x_1, x_2, \ldots$$

in X. Choose also $x \in X$.

We say that this sequence is *convergent to* x if, for every $\epsilon > 0$, there exists $N > 0$ so that

$$i > N \implies d(x_i, x) < \epsilon.$$

Write, exploring what this definition means. You might want to take $(X, d) = (\mathbb{R}, d_{std})$ if that helps. You might also consider playing with examples, like

- The sequence given by $x_1 = 1, x_2 = 1/2, x_3 = 1/3, \ldots, x_i = 1/i$ in $(\mathbb{R}, d_{std})$. How about the same sequence, but in $(\mathbb{R}, d_{discrete})$?
- The sequence given by $x_1 = (\cos(1), \sin(1)), x_2 = (\cos(2)/2, \sin(2)/2), x_3 = (\cos(3)/3, \sin(3)/3, \ldots, x_j = (\cos(j)/j, \sin(j)/j), \ldots$. Does this sequence converge in $(\mathbb{R}^2, d_{std})$?

Some questions you might ask include:

1. Why should I care about this definition? Is it useful?
2. Can I think of examples?
3. What does this have to do with continuity, if anything?
4. How important is the metric?

As usual, I want to see your thinking, but you must write clearly, and in a way where I can see your thinking. When you are being imprecise, acknowledge it, so you know when you are being a bit poetic.