

Writing assignment due Thursday, September 19

Let (X, d_X) and (Y, d_Y) be metric spaces, and fix a function $f : X \rightarrow Y$. We have learned that the following three properties of f are equivalent:

1. f is continuous. Remember, this means that for every $x \in X$ and for every $\epsilon > 0$, there exists $\delta > 0$ so that

$$d_X(x, x') < \delta \implies d_Y(f(x), f(x')) < \epsilon.$$

2. The preimage of any open set in Y is an open set in X .
3. f preserves convergence of sequences. That is, if x_1, x_2, \dots is a (infinite) sequence in X that converges to $x \in X$, then $f(x_1), f(x_2), \dots$ is a sequence in Y that converges to $f(x)$.

Remark 3.8.1. Recall that by ‘equivalent,’ we mean that any one of the above properties implies all others. For example, 1 implies 2 (and 3), while 2 implies 1 (and 3).

Explore and think, and then write, about why one might expect, or come to believe, that the above three properties are equivalent. You will want to draw upon the definitions of open set and of convergent sequence. You may not have time to relate all three properties; but the important thing is for you to think deeply and precisely, and to express your thoughts well.

I do not want to give you a time limit or a recommended time. But this is your chance to just think through these ideas, so take your time.