

## Writing due Thursday, September 26

**Preliminaries.** Fix a set  $X$ . Recall that an *equivalence relation* on  $X$  is a choice of subset

$$E \subset X \times X$$

satisfying the following properties:

1. (Reflexivity) For every  $x \in X$ , we have that  $(x, x) \in E$ .
2. (Symmetry) For every  $x, x' \in X$ , we have that if  $(x, x') \in E$ , then  $(x', x) \in E$ .
3. (Transitivity) For every  $x, x', x'' \in X$ , we have that if  $(x, x') \in E$  and  $(x', x'') \in E$ , then  $(x, x'') \in E$ .

As a matter of notation, we write  $x \sim x'$  if and only if  $(x, x') \in E$ .

An *equivalence class* of  $E$  is a non-empty subset  $A \subset X$  such that (i) If  $x \in A$  and  $x \sim x'$ , then  $x' \in A$ , and (ii) If  $x, x' \in A$  then  $x \sim x'$ .

Given an equivalence relation  $E$  on  $X$ , and an element  $x \in X$ , we write  $[x]$  for the (unique) equivalence class containing  $x$ .

Given an equivalence relation  $E$ , we denote by  $X/\sim$  the set of equivalence classes of  $E$ .

Note that this definition has nothing to do with topological spaces; it is just a way to construct a new set  $X/\sim$  out of the data of an equivalence relation on  $X$ .

**The assignment.** For this week, I want you to spend at least an hour thinking about the notion of an equivalence relation and the set  $X/\sim$ . It may be confusing that  $X/\sim$  is a set of sets, but  $X/\sim$  is supposed to capture the notion of “the set you get by identifying elements of  $X$  if they are related by  $E$ .”

Might there be a different way to define something capturing this notion? Why might equivalence relations and sets like  $X/\sim$  come up in mathematics? Can you find examples? Write away, and remember to distinguish when you are being (im)precise.

Finally, I will remind you that the goal of this writing assignment is *not* for you to try to convince me that you understand. I rather want you to write honestly about what you are exploring, so that I can see what you are *thinking*. If you do not show a sufficient amount of thought and exploration, you will not get a high grade.