Extra Credit Assignment 3

Due Thursday, September 17, 11:59 PM

Sometimes, an object is determined by understanding the functions *in*to it, or the functions *out* of it.

Let P and Q be posets, and let $P \times Q$ be the product poset.

(a) Prove that the *projection* maps $P \times Q \to P$ and $P \times Q \to Q$ are both maps of posets. (These are the maps defined by $(p, q) \mapsto p$ and $(p, q) \mapsto q$, respectively.)

(b) Let W be another poset. Prove that for any poset map $W \xrightarrow{a} P \times Q$, the compositions

$$W \xrightarrow{a} P \times Q \to P, \qquad W \xrightarrow{a} P \times Q \to Q$$

are both maps of posets. (Here, the maps out of $P \times Q$ are the projection maps.)

(c) Let $\hom(W, P \times Q)$ denote the set of all *poset maps* from W to $P \times Q$. Likewise, $\hom(W, P)$ and $\hom(W, Q)$ are the set of poset maps from W to P, and from W to Q, respectively. Prove that composition by the projection maps induces a function

$$j : \hom(W, P \times Q) \to \hom(W, P) \times \hom(W, Q).$$

(d) Prove that j is a bijection.

In words, this assignment is saying that "a poset map to $P \times Q$ is the exact same thing as the data of a poset map to P, and a poset map to Q." This is called the *universal property of the product.*