Extra Credit Assignment 4

Due Thursday, September 24, 11:59 PM

Part I

Let X be a set with n elements in it. How many different topologies does X admit?

Concretely, how many subsets $\mathcal{T} \subset \mathcal{P}(X)$ satisfy the properties of being a topology?

For example, if $n = 1$, then X admits one topology.

If $n = 2$, X admits four topologies.

Tell me the answer for $n = 1, 2, 3, 4$, and 5. Bonus points if you can tell me for $n = 6$ and $n = 7$.

Part II

Let's say that two topologies $\mathcal{T}$ and $\mathcal{T}'$ on X are called *equivalent* if there is a bijection $f : X \to X$ such that the following holds: The function $\mathcal{P}(X) \to \mathcal{P}(X)$ sending a subset A to $f^{-1}(A)$ sends any $U \in \mathcal{T}$ to $f^{-1}(U) \in \mathcal{T}'$, and moreover, the resulting function

$$\mathcal{T} \to \mathcal{T}', \qquad U \mapsto f^{-1}(U)$$

is a bijection.

How many *inequivalent*, aka non-equivalent, topologies does a set X with n elements admit? Tell me the answer for $n = 3, 4, 5$, and bonus if you can tell me for $n = 6, 7$.

For example, if $n = 1$, then X admits one topology.

If $n = 2$, X admits three non-equivalent topologies.