Extra Credit Assignment 5

Due Thursday, October 1, 11:59 PM

This extra credit assignment has many sections; pick and choose which sections you want to do. I recommend Section 1 at the least, as it serves as a review of some things you have learned.

Section 1. Terms and types

Below is a bank of terms, and a bank of *types*. For each of the terms, tell me what type or types of objects the term is allowed to describe.

Term bank:

(i) Continuous

(ii) Topology

(iii) Partial order relation

(iv) Compact

(v) Open

(vi) Closed

(vii) Bounded

(viii) Converges

Type bank:

(A) A number

(B) A function between two sets

(C) A function between two topological spaces

(D) A subset of the power set of a set X.

(E) A subset of $X \times X$, where X is some set.

(F) A subset of a topological space.

(G) A sequence in $\mathbb{R}^n$.

(H) A distance between two points in $\mathbb{R}^n$

(I) A topological space.

Your answers could look, for example, simply like

- (i) F
- (ii) A, B, C

- (iii) G
- (iv) A, H, F

  et cetera.

Section 2. Convergence and continuity. Let us define

$$Z := \mathbb{Z}_{>0} \bigcup \{\infty\}.$$

In other words, Z is the set consisting of the positive integers, and one more element that we call "∞." We will define a topology on Z as follows:

A subset $U \subset Z$ is open if and only if one of the following holds:

1. $\infty \notin U$, or
2. $\infty \in U$ and the complement of U is finite.

I will leave the verification that this is a topology on Z to you. (By the way, it may help to remember that the empty set is a finite set.)

Section 2 Part I. Now let $Z \to \mathbb{R}^n$ be a function. We will denote the value of this function on $i \in \mathbb{Z}_{>0}$ by a_i, and the value on $\infty \in Z$ by a_∞. Show that a function $Z \to \mathbb{R}^n$ is continuous if and only if the sequence $a_1, a_2, \ldots$ converges to a_∞.

Section 2 Part II. Now show that a function $f : \mathbb{R}^m \to \mathbb{R}^n$ is continuous if and only if the following holds: For every sequence $a_1, a_2, \ldots$ in $\mathbb{R}^m$ that converges to b, the sequence $f(a_1), f(a_2), \ldots$ in $\mathbb{R}^n$ converges to $f(b)$.