Extra Credit Assignment 7 (Blood types and posets)

Due Thursday, October 15, 11:59 PM

This is an extra credit assignment about posets.

As you may know, blood types affect whether we can safely supplement a patient with blood-derived resources from another patient. For example, if you wanted to donate red blood cells to another person, your blood type needs to be compatible with the recipient's. If you wanted to donate your plasma, you must likewise check for compatibilities.[5]

Roughly speaking, the four main blood types that humans can have are O, A, B, AB. These are distinguished by certain markers (called antigens) on the surfaces of red blood cells. We can ask not just for this ABO classification of blood types, and look for another kind of antigen, called RhD, and append a *positive* or a *negative* to the blood type based on whether this RhD factor is present.

Using ABO and RhD antigens, we can divide human blood types into the following eight types:

$$O-, O+, A-, A+, B-, B+, AB-, AB+$$

(We can, of course, keep classifying blood types by recording more and more antigens on red blood cells. We would probably only pursue further classification if it were clinically useful, but I'm not an expert. According to Wikipedia, there are up to 38 recognized human blood groups. The above four, and the above eight, are the most commonly known and used classifications.)

[5]By the way, it's good to know the blood types of yourself and your loved ones—not just in case of emergencies, but in case there is a call for donations of particular blood types. If you or are a loved one are pregnant, your blood time will probably be tested to make sure there is compatibility with the baby's blood type.

Below is a table showing blood type compatibilities for red blood cell transfusions. A check mark indicates that a donor having the blood type

		Donor blood type							
		O-	O+	A-	A+	B-	B+	AB-	AB+
Recipient blood type	O-	✓	×	×	×	×	×	×	×
	O+	✓	✓	×	×	×	×	×	×
	A-	✓	×	✓	×	×	×	×	×
	A+	✓	✓	✓	✓	×	×	×	×
	B-	✓	×	×	×	✓	×	×	×
	B+	✓	✓	×	×	✓	✓	×	×
	AB-	✓	×	✓	×	✓	×	✓	×
	AB+	✓	✓	✓	✓	✓	✓	✓	✓

Table 1: Compatibility for red blood cell transfusions

in the same column as the check mark can safely transfuse red blood cells to a recipient having the blood type in the row of the check mark. So for example, someone with blood type $A-$ can safely donate red blood cells to somebody with blood type $A+$.

Define

$$P := \{O-, O+, A-, A+, B-, B+, AB-, AB+\}.$$

So P is an eight-element set consisting of the eight blood types discussed above. Now we define a subset $R \subset P \times P$ as follows. Given two elements $p, p' \in P$, we say that $(p, p') \in R$ if and only if a donor of type p can safely give a red blood cell transfusion to a recipient of type p'. So for example, according to the above table, $(O-, O+) \in R$, but $(O+, O-)$ is not an element of R.

Question One. Is P, endowed with this relation, a poset?

		Donor			
		O	A	B	AB
Recipient	O	✓	✓	✓	✓
	A	×	✓	×	✓
	B	×	×	✓	✓
	AB	×	×	×	✓

Table 2: Compatibility for plasma transfusions

Below is a table showing blood type compatibilities for plasma transfusions. Note that the compatibilities are different based on whether you want to study red blood cell transfusions or plasma transfusions. Note also that here, we do not care about the RhD antigen (so we ignore the plus and minus types).

Define

$$Q = \{O, A, B, AB\}.$$

And define a relation $R_Q \subset Q \times Q$ by declaring that $(q, q') \in R_Q$ if and only if someone of blood type q can safely donate plasma to someone of blood type q'.

Question 2. Is R_Q a partial order relation on Q?

Now, note that there is a "forgetful" map

$$f : P \to Q$$

which forgets the RhD type of a blood type. So for example, f sends $AB-$ to AB.

Question 3. Does it make sense to ask whether f is a map of posets? If so, is f a map of posets?

Finally, given a poset T, we may construct another poset called T^{op}, defined as follows: T^{op} has the same elements as T, but we declare our relation so that $(t, t') \in R_{T^{\text{op}}}$ if and only if $(t', t) \in R_T$, where R_T is the partial order relation for T.

Question 4. Let $g : P \to Q^{\text{op}}$ be the function that forgets the RhD type of a blood type. Does it make sense to ask whether g is a map of posets? If so, is g a map of posets?