Homework 5

Due Tuesday, September 29, 11:59 PM

Proof problem

Let m and n be integers with both m and n at least 1. Let $X = \mathbb{R}^m$ and $Y = \mathbb{R}^n$, and fix a function $f : X \to Y$. We give both X and Y the standard topology of Euclidean space.

The problem. Show that the following two statements are equivalent:

(a) For any open subset $V \subset Y$, the preimage $f^{-1}(V)$ is open.

(b) For every $x \in X$ and for every real number $\epsilon > 0$, there exists a real number $\delta > 0$ such that

$$x' \in \mathrm{Ball}(x, \delta) \implies f(x') \in \mathrm{Ball}(f(x), \epsilon).$$

As you do this problem, it may help to note that $\mathrm{Ball}(x, \delta) \subset X$, while $\mathrm{Ball}(y, \epsilon) \subset Y$.

Canvas Questions:

For each of the following subsets A of $\mathbb{R}^2$, you should be able to tell me whether the subset is

(i) Closed but not bounded

(ii) Closed and bounded and compact

(iii) Bounded but not closed

(iv) Not closed and not bounded

(v) Closed and bounded but not compact

Here are the subsets:

(a) $A = \Delta^1$.

(b) A is the open ball of radius 3 inside $\mathbb{R}^2$, centered at the point $(1, 1)$.

(c) A is a set consisting of exactly three points in $\mathbb{R}^2$. (I have not told you which three points.)

(d) $A = S^1$.

(e) $A = \mathbb{R}^2$.

(f) $A = D^2 \setminus \{(0, 0)\}$. In other words, A is obtained by removing the origin from D^2.