Homework 6

Due Tuesday, October 6, 11:59 PM

Write down the definition, or complete the definition, of each of the following terms. You will be required to know these for the exam. (There are more terms you may need to know, by the way.)

(1) Topology on a set X.

(2) If X and Y are topological spaces, and $f : X \to Y$ is a function, f is called *continuous* when....?

(3) The Alexandroff topology on a poset P.

(4) The standard topology on $\mathbb{R}^n$.

(5) The discrete topology on a set X.

(6) An open subset of an arbitrary topological space X. (That is, X need not be $\mathbb{R}^n$, nor a poset.)

(7) A closed subset of a topological space X.

(8) A cover of a set X.

(9) An open cover of a topological space X.

(10) A topological space is *compact* if...?

Canvas Questions:

(a) Let $f : X \to Y$ be a function, and suppose X and Y are topological spaces with topologies $\mathcal{T}_X$ and $\mathcal{T}_Y$, respectively.

Which of the following is equivalent to the condition that f be continuous? (Check all that apply.)

1. For any open $U \subset Y$, we have that $f^{-1}(U)$ is an open subset of X.
2. For any $V \subset Y$, we know that $V \in \mathcal{T}_Y \implies f^{-1}(V) \in \mathcal{T}_X$.

3. Whenever $B \subset Y$ is open, we know that the preimage of B is open.

4. Whenever $B \subset Y$ is closed, we know that the preimage of B is closed.

5. Whenever $U \subset X$ is open, we know that $f(U) \subset Y$ is open.

(b) Let X be a topological space. Which of the following implies that a subset $B \subset X$ is closed? (Check all that apply.)

1. $B = X$.

2. $B = \emptyset$.

3. B is an intersection of finitely many closed sets.

4. B is an intersection of closed sets.

5. B is a union of finitely many closed sets.

6. B is a union of closed sets.

7. B is a union of finitely many open sets.

8. B has infinitely elements in it.

9. B has finitely many elements in it.

10. The complement of B is open.

(c) Let $X = \mathbb{R}^n$ with the standard topology. Which of the following implies that a subset $B \subset X$ is closed? (Check all that apply.)

1. $B = X$.

2. $B = \emptyset$.

3. B is an intersection of finitely many closed sets.

4. B is an intersection of closed sets.

5. B is a union of finitely many closed sets.

6. B is a union of closed sets.

7. B is a union of finitely many open sets.

8. B has infinitely elements in it.
9. B has finitely many elements in it.
10. The complement of B is open.