Homework 7

Due Tuesday, October 13, 11:59 PM

Proof problem

Let X be a set and $\sim$ an equivalence relation on X. Let $q : X \to X/\sim$ be the function $x \mapsto [x]$.

Prove that the quotient set $X/\sim$ satisfies the following universal property:

(i) For any other set Z, and any function $f : X \to Z$ satisfying the property that $x \sim x' \implies f(x) = f(x')$, there exists a function $f' : X/\sim \to Z$ for which

$$f' \circ q = f.$$

(ii) Moreover, this function f' is the unique function satisfying the property above. That is, if g is any other function satisfying $g \circ q = f$, then in fact $g = f'$.

Canvas True/False Questions:

In the following, let X be a set.

True or False: Any partial order relation on X is an equivalence relation on X.

True or False: Any equivalence relation on X is a partial order relation on X.

True or False: If $R \subset X \times X$ is both an equivalence relation and a partial order relation, then R must be the diagonal of X. (That is, $(x, x') \in R \iff x = x'$.)

True or False: Let $\sim$ be an equivalence relation on X. Then the projection map $p : X \to X/\sim$ is a surjection.

True or False: Let $\sim$ be an equivalence relation on X, and let $p : X \to X/\sim$ be the projection map. If p is an injection, then $\sim \subset X \times X$ is the diagonal of X.

True or False: Let $\sim$ be an equivalence relation on X. Then for any $x \in X$, we have that $[x] = X$.