Homework 8

Due Tuesday, October 20, 11:59 PM

Proof problem

(a) State the definitions of:

  (i) Equivalence relation

  (ii) Equivalence class

  (iii) Quotient set

  (iv) Quotient map

  (v) Quotient topology (given a topological space X and an equivalence relation on X).

(b) Suppose X is a compact topological space. Prove that for any equivalence relation $\sim$ on X, the quotient space $X/\sim$ is compact.

(c) Fix an equivalence relation $\sim$ on a topological space X. Suppose that $f : X \to Y$ is a continuous map with the property that if $x \sim x'$, then $f(x) = f(x')$.

  (i) Construct a function $f' : X/\sim \to Y$ so that $f' \circ q = f$.

  (ii) Prove that f' is continuous.

Canvas True/False Questions:

Indicate whether each of the following statements is true or false:

1. If $\sim$ is an equivalence relation on a set X, then the quotient map $X \to X/\sim$ is a surjection.

2. If $\sim$ is an equivalence relation on a topological space X, then the quotient map $X \to X/\sim$ is a surjection.

3. If $\sim$ is an equivalence relation on a topological space X, then the quotient map $X \to X/\sim$ is continuous.

4. Define an equivalence relation on $\mathbb{R}$ as follows: We say that $x \sim x'$ if and only if there exists a *non-zero* real number t for which $tx = x'$. Then $\mathbb{R}/\sim$ is compact. (Here, we give $\mathbb{R}$ the standard topology.)

5. Define an equivalence relation on $\mathbb{R}^2 \setminus \{(0,0)\}$ as follows: We say that $(x_1, x_2) \sim (x_1', x_2')$ if and only if there exists a non-zero real number t for which $(tx_1, tx_2) = (x_1', x_2')$. Then $(\mathbb{R}^2 \setminus \{(0,0)\})/\sim$ is compact.

6. Define an equivalence relation on S^1 as follows: We say that $(x_1, x_2) \sim (x_1', x_2')$ if and only there exists a non-zero real number t for which $(tx_1, tx_2) = (tx_1', tx_2')$. Then $S^1/\sim$ is compact.