Homework 9

Due Tuesday, October 27, 11:59 PM

Proof problem

(a) State:

(i) The definition of the *direct product* $X \times Y$ of two sets X and Y. (Otherwise known as the Cartesian product.)

(ii) The definition of the *product topology* on $X \times Y$ (when X and Y are topological spaces themselves).

(iii) The universal property of the direct product $X \times Y$ of spaces.

(iv) The definition of the *direct product* $\prod_{\alpha \in \mathcal{A}} X_\alpha$ of (possibly infinitely many) sets $\{X_\alpha\}_{\alpha \in \mathcal{A}}$.

(v) The universal property of the direct product $\prod_{\alpha \in \mathcal{A}} X_\alpha$ of (possibly infinitely many) spaces .

(b) Let $\mathbb{R}$ be given the standard topology. Show that the product topology on $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$ is the same topology as the standard topology on $\mathbb{R}^2$.

(c) Equip $\mathbb{R}^2$ with the standard topology and let $p_1 : (x_1, x_2) \mapsto x_1$, $p_2 : (x_1, x_2) \mapsto x_2$ be the two projection maps to $\mathbb{R}$. Let X be a topological space. Prove that a function $f : X \to \mathbb{R}^2$ is continuous if and only if both $p_1 \circ f$ and $p_2 \circ f$ are continuous.

Canvas True/False Questions:

Indicate whether each of the following statements is true or false:

1. If X and Y are finite sets, then $X \times Y$ is a finite set.

2. If X and Y are two spaces with the trivial topology, then the product topology on $X \times Y$ is the trivial topology.

3. If X and Y are two spaces with the discrete topology, then the product topology on $X \times Y$ is the discrete topology.

4. If X and Y are two spaces, and if the product topology on $X \times Y$ is the trivial topology, then X and Y are spaces with trivial topologies.

5. If X and Y are two spaces, and if the product topology on $X \times Y$ is the discrete topology, then X and Y are spaces with discrete topologies.

6. If $X = \emptyset$ and Y is any set, then $X \times Y$ is the empty set.