Homework 10

Due Tuesday, November 3, 11:59 PM

Proof problem

(a) State:

  (i) The definition of what it means for a topological space X to be Hausdorff.

  (ii) The definition of a metric on a set X.

  (iii) The definition of a sequence in a metric space converging to a point.

  (iv) The definition of an open ball of radius r centered at a point x in a metric space.

  (v) The definition of the metric topology on a metric space.

(b) Let (X, d_X) and (Y, d_Y) be metric spaces. Fix a function $f : X \to Y$. Show that the following are equivalent.

  (1) f is continuous (when giving X and Y the metric topology).

  (2) For every $\epsilon > 0$ and for every $x \in X$, there exists a $\delta > 0$ so that

$$d_X(x, x') < \delta \implies d_Y(f(x), f(x')) < \epsilon.$$

  (3) For every sequence $(a_i)_{i \geq 1}$ in X converging to a, the sequence $(f(a_i))_{i \geq 1}$ in Y converges to $f(a)$.

Canvas True/False Questions:

Indicate whether each of the following statements is true or false:

1. Every metric space is Hausdorff.
2. If X is Hausdorff, then for any equivalence relation on X, $X/\sim$ is Hausdorff.
3. Euclidean space (with the standard topology) is Hausdorff.

4. If P is a poset, P (with the Alexandroff topology) is Hausdorff if and only if P is discrete.

5. If X and Y are Hausdorff, then $X \times Y$ is Hausdorff.

6. If at least one of X or Y is not Hausdorff, then $X \times Y$ is not Hausdorff.