Homework 11

Due Tuesday, November 10, 11:59 PM

Proof problem

(a) State:

(i) What it means for a sequence $a_1, a_2, \ldots$, in a metric X space to converge to a point b.

(ii) The definition of an isometry between two metric spaces.

(iii) The definition of the taxi cab metric on $\mathbb{R}^n$.

(iv) The definition of the l^∞ metric on $\mathbb{R}^n$.

(v) The definition of the discrete metric on a set X.

(b) Draw the open ball of radius 1 in $\mathbb{R}^2$, centered at the origin, with respect to the l^∞ metric.

(c) Draw the open ball of radius 1 in $\mathbb{R}^2$, centered at the origin, with respect to the taxi cab metric.

(d) Show that the metric topologies induced by the standard metric, the taxicab metric, and the l^∞ metric are all equal.

Canvas True/False Questions:

Below, X and Y are metric spaces. Indicate whether each of the following statements is true or false:

1. If X is a metric space with the discrete metric, then the induced metric topology is the discrete topology.

2. Any isometry is a homeomorphism. (More precisely: If $f : X \to Y$ is an isometry between metric spaces, then it is also a homeomorphism when endowing X and Y with their metric topologies.)

3. Any homeomorphism is an isometry. (More precisely: If $f : X \to Y$ is a map between metric spaces for which f is a homeomorphism with respect to the metric topologies, then f is an isometry.)

4. Let $f : X \to Y$ be a map between metric spaces such that for all $x, x' \in X$, we have that $d(f(x), f(x')) = d(x, x')$. Then f is an injection.

5. Suppose $f : X \to Y$ is a function satisfying the following property: There exists a real number $c > 0$ so that for every $x, x' \in X$, $d(f(x), f(x')) \leq cd(x, x')$. Then f is continuous.

6. Let $d : X \times X \to \mathbb{R}$ be a metric. Then for any subset $A \subset X$, the composition of d with the inclusion $A \times A \to X \times X$ is a metric on A.