Homework 12

Due Tuesday, November 17 , 11:59 PM

Definitions

State:

(i) The definition of a (continuous) path in a space X.

(ii) The definition of path-connectedness.

(iii) The definition of $\pi_0(X)$.

(iv) The Invariance of Domain theorem.

(v) The definition of connectedness.

Proof(s)

(a) Prove that if X and Y are homeomorphic, then there is a bijection between $\pi_0(X)$ and $\pi_0(Y)$.

(b) Exhibit an example showing that the converse is false.

Canvas True/False Questions:

Indicate whether each of the following statements is true or false:

1. If X is path-connected, then $\pi_0(X)$ consists of a single point.
2. If $\pi_0(X)$ consists of a single point, then X is path-connected.
3. Let $A \subset \mathbb{R}^2$ be a finite subset. Then $\mathbb{R}^2 \setminus A$ is path-connected.
4. Let $A \subset \mathbb{R}$ be a finite subset containing n elements. Then $\pi_0(\mathbb{R} \setminus A)$ consists of $n + 1$ elements.
5. Suppose $n \geq 1$ and let $A = S^{n-1} \subset \mathbb{R}^n$. Then $\mathbb{R}^n \setminus A$ is path-connected.
6. Suppose $n \geq 2$ and let $A = D^n \subset \mathbb{R}^n$. Then $\mathbb{R}^n \setminus A$ is path-connected.