Homework 14

Due Tuesday, December 1, 11:59 PM

Definitions

State:

(i) The definition of a topology.

(ii) The definition of a continuous function between topological spaces. (If your answer involves epsilons or deltas, you're barking up the wrong tree.)

(iii) The definition of homeomorphism.

(iv) The definition of compactness.

(v) The definition of what it means for a space to be Hausdorff.

(vi) The definition of the quotient topology on $X/\sim$.

(vii) The definition of the product topology on $X \times Y$.

(viii) The definition of a metric.

(ix) The definition of the standard topology on $\mathbb{R}^n$.

(x) The definition of the closure of a subset of a topological space X.

Theorems/Results/Examples

(a) State:

(i) The Heine-Borel theorem.

(ii) The invariance of domain theorem.

(iii) The extreme value theorem (for continuous functions to $\mathbb{R}$ whose domains are compact).

(iv) The universal property of the quotient topology.

(v) The universal property of the subspace topology.

(vi) Three equivalent characterizations for a function between metric spaces to be continuous.

(vii) The Brouwer Fixed Point Theorem.

(viii) An example of a compact topological space.

(ix) An example of a topological space that is not path-connected.

(x) An example of a non-compact topological space.

(xi) An example of a subset of $\mathbb{R}^2$ that is closed.

(xii) An example of a subset of $\mathbb{R}^2$ that is open.

(xiii) An example of a subset of $\mathbb{R}^2$ that is both closed and open.

(xiv) An example of a subset of $\mathbb{R}^2$ that is neither closed nor open.