Writing Assignment 3

Due Friday, September 11, 11:59 PM

The goal of this assignment is to get a feel for what a reader (such as yourself!) should want out of a written proof; after all, you'd like to read a proof that demonstrates why a statement is true.

In doing this assignment, I'm hoping you'll also get a feel for how *you (as a reader) would want* a proof to be written; this could help guide you in your writing proofs for this class. (You should emulate the writing you want to read!)

Instructions. Below are past submissions to Homework 01, parts (a) and (b). Take a look. (They are copied from specific submissions, but they are representative of many of the submissions of your peers.)

Part I (at most 30 minutes). Go back and read the original statements of (a) and (b) from Homework 01. In as clear language as you can muster, write down what you are supposed to prove in part (a), and what you are supposed to prove in part (b). For some of you, this part may take a few minutes, and for others, longer—that's totally normal and depends on how well Math 3330 was taught to you; but do not spend more than 30 minutes on this. If you feel stuck, you should e-mail Hiro and seek help.

Part II (about 60 minutes). *Choose one* of the four submissions that Hiro pasted into this document (see the next pages)—either option (a)(1), (a)(2), (b)(1), or (b)(2). Read through it. Do you understand the proof? Is the desired statement proven? If something is unclear, can you tell whether that is due to the writing, or to the reading? Keep in mind that this exercise is not meant to be a personal critique of either yourself or of the writing; rather, this is meant to give you *practice* in evaluating the scientific and mathematical content of a piece of writing (the sample). Write down if there are any parts of the writing that you feel are accurate, precise, and helpful, or if there are parts of the writing that you feel otherwise about; write to me about any parts of the writing that cause you to be unsure whether it is the reader's (your) comprehension, or the writing's content, that is preventing a proof from being understood. In your writing, *tell me which option you chose, and make sure to be as specific as possible* throughout your writing, especially when referencing specific words or symbols.

Option (a)(1):

a) Define $\phi : P(X) \to \{0,1\}^X$
$\phi(A) = f_A$

: Let $\phi(A) = \phi(B) \Rightarrow f_A = f_B \Rightarrow A = B \Rightarrow \phi$ ie 1-1

Now let $g \in \{0,1\}^X$ ie. $g : X \to \{0,1\}$
$T = \{x \in X : g(x) = 1\}$
$T \subset X \Rightarrow T \in P(X)$, we can define
$f_T : X \to \{0,1\}$

$$f_T(x) = \begin{cases} 1 & ; x \in T \\ 0 & ; x \notin T \end{cases}$$

$\Rightarrow g = f_T \Rightarrow \phi(T) = f_T = g$

$\Rightarrow \phi$ ie. onto

$\Rightarrow P(X) \sim \{0,1\}^X \Rightarrow |P(X)| = 2^{|X|}$

Option (a)(2):

(a) Exhibit a bijection between $P(A)$ and the set of all functions from A to two element set $\{0,1\}$

A bijection f can be defined as follows
For each $B \in P(A)$, let $f(B) : A \to \{0,1\}$ be defined so that $f(B)(a) = 1$ if $a \in B$ and $f(B)(a) = 0$ if $a \notin B$

Proof: Let C be the set of functions from A into $\{0,1\}$
Let $g : C \to P(A)$ be given by $g(h) = \{a \in A \mid h(a) = 1\}$
For each $B \in P(A)$, $g(f(B)) = \{a \in A \mid f(B)(a) = 1\} = B$
For each $h \in C$ and $a \in A$, $f(g(h))(a) = f(\{a \in A \mid h(a) = 1\})(a)$
$= h(a)$, so $f(g(h)) = h$
$\therefore$ f and h are inverses, so f is a bijection ■

Option (b)(1):

Problem. *Prove that there is no bijection between A and $P(A)$.*

Proof. Suppose there is a bijection θ from A onto $P(A)$. Let

$$X = \{f \mid f : A \to \{0,1\}\}.$$

We know there is a bijection between $P(A)$ and X as established by the previous problem. And by assumption we know there is a bijection θ between A and $P(A)$. Hence, there is a bijection ϕ between A and X. Define $\beta : A \to \{0,1\}$ to be the function such that for each $a \in A$,

$$\beta(a) \neq \phi(a)(a).$$

I want to show that $\beta \notin \phi(A)$. Suppose there exists $a \in A$ such that $\phi(a) = \beta$. Then, $\phi(a)(a) = \beta(a)$. This is a contradiction. Thus, $\beta \notin \phi(A)$. Thus, ϕ is not surjective. This means that ϕ is not a bijection. Thus, there is no bijection between A and X which implies that there is no bijection between A and $P(A)$. □

Option (b)(2):

Let $f : A \to P(A)$ then if $x \in A$

$\Rightarrow f(x) \in P(A)$

$\Rightarrow f(x) \subset A$

either $x \in f(x)$ or $x \notin f(x)$

Define $X = \{x \in A \mid x \notin f(x)\}$

$\Rightarrow X \subset A$

$X \in P(A)$

$f : A \to P(A)$ cannot be onto

A is any non empty set

from A to $P(A)$, onto function can't be defined.