Writing Assignment 4

Due Friday, September 18, 11:59 PM

Some students learn what a continuous function is as the following: "A function is continuous if its graph can be drawn without ever lifting your pen."

This of course is kind of false, because something like $\tan(x)$ is continuous everywhere it's defined, but clearly you can't draw the graph of $\tan(x)$ without every lifting your pen. (Try it!)

Regardless, you may have also learned the "epsilon-delta," or "ϵ-δ" definition of continuity as well: That a function $f : \mathbb{R} \to \mathbb{R}$ is called continuous if for every $x \in \mathbb{R}$, and for every $\epsilon > 0$, there exists $\delta > 0$ so that

$$|x - x'| < \delta \implies |f(x) - f(x')| < \epsilon.$$

Note that in this assignment, we are *not* using any notion of open balls or topological space. This is a writing assignment you could have received before ever taking this course, and this is how I would like you to treat it.

Prompt. Explain to me how, or if, this intuition of "never lifting your pen" (which is a non-mathematical, imprecise notion) agrees with, or is compatible with, the "ϵ-δ" definition for compatibility.

This is another assignment that takes a long time. I would suggest one hour just exploring the prompt, especially making sure you understand the ϵ-δ definition (trying out examples of different f—like $f(x) = x$, and $f(x) = x^2$— and different values of ϵ—like $\epsilon = 1, 0.1, 0.01, \ldots$—may be helpful).

Then, spend the next half hour writing.