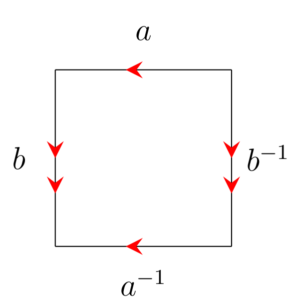
Writing Assignment 14

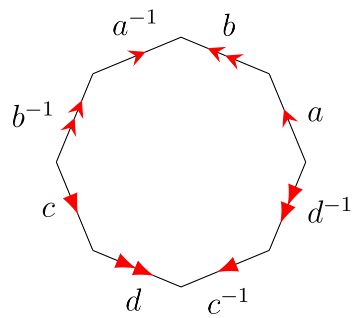
Due Monday, November 30, 11:59 PM

Below is an image of a (solid) square, otherwise known as the space $[0,1] \times [0,1]$. Drawn on the square are also instructions on how to glue certain edges. For example, the top horizontal edge, labeled a, is to be glued to the bottom horizontal edge, labeled a^{-1}. The gluing is to be done so that the directions indicated are preserved; so for example, the rightmost point of a (the upper-right corner of the square) is to be glued to the rightmost point of a^{-1} (the bottom-right corner of the square), while the leftmost point of a (the upper-left corner of the square) is to be glued to the leftmost point of a^{-1} (the bottom-left corner of the square). More generally, every point (x, y) along the edge a is to be glued to the corresponding point on the edge a^{-1} with the same x-coordinate. This gluing *preserves* the indicated directionality of the arrows.

Likewise drawn, using arrowheads, are gluing instructions for how to glue the edge b to the edge b^{-1}. The reason that b has two arrowheads and a only has one is to remind us that b edges are not glued to a-edges.

Part I. Drawing as carefully and as accurately as you can, describe why the resulting shape is a torus (the surface of a doughnut), otherwise known as the surface of genus one.





Above is an image of an octagon. Also labeled are instructions on how to glue edges. The edges labeled a and a^{-1} are to be glued to each other. So are the edges labeled c and c^{-1}. Likewise, we glue the edge b to the edge labeled b^{-1}, and likewise for d and d^{-1}.

We again glue in such a way that the orientations indicated by the arrows are respected. So perhaps the "a^{-1}" notation is more clear–though a is oriented *counter*clockwise, we glue a to a^{-1} using the clockwise orientation for the edge a^{-1}.

Part II. Drawing as carefully and as accurately as you can, describe why the resulting shape is a two-holed torus (the surface of a two-person tube, without the plastic handles), otherwise known as the surface of genus two.