4 Writing Assignment Due Thursday, February 20

We are re-visiting the ϵ-δ definition of limit, because have not yet shown sufficient thought and understanding! **Spend at least two hours thinking and exploring** before you even begin to write your assignment.

Prompt. Your friend is learning the ϵ-δ definition of a limit. They are confused beyond belief. They may ask questions like:

1. What is L? How should I think about it?
2. What is ϵ? How should I think about it? Why do I need a δ for "every" ϵ?
3. What is δ? How should I think about it? Is there one δ that fits for every ϵ? Or do I need a possibly different δ to exist for every ϵ?
4. Am I allowed to change this definition?
5. How does the word "limit" even come into this definition?

Write an explanation of the ϵ-δ definition to help your friend understand *what* this definition is saying, and *how* the definition might conform to the intuition of a limit. Some intuitions we are supposed to have include: "the limit $\lim_{x\to a} f(x)$ is the value that f seems to 'want' to take at a," or "the limit is the value that f 'approaches' as x approaches a."

At the very least, it will help if you carefully copy the epsilon-delta definition of limit.

Some guidelines. You *must* think for a long time to even begin to understand. I will repeat what I said above: **Spend at least two hours thinking and exploring** before you even begin to write your assignment.

Format. See online. Only PDF uploads are accepted on Canvas.

Reminder. Let f be a function. We say that f *has a limit at* a if there is a number L satisfying the following property:

For every $\epsilon > 0$, there exists a $\delta > 0$ so that

$$0 < |x - a| < \delta \implies |f(x) - L| < \epsilon.$$

That is, so long as $|x - a| < \delta$ and $x \neq a$, then $|f(x) - L| < \epsilon$.

Finally, we call the number L the *limit of f as x approaches a*, and we write

$$\lim_{x \to a} f(x) = L.$$

Some exploratory hints: $|x - a|$ measures the distance between x and a. What does it means for this distance to be less than δ? Likewise, $|f(x)-L|$ measures the distance between $f(x)$ and L. Also, if you like, you can replace the phrase “there exists” with the phrase “we can find.” You may also replace the phrase “for every” with “regardless of our choice of.”

Some context. This is the *hardest* definition in calculus. It takes a lot of thought to come to grips with this definition, and most calculus students do not understand this definition. I want you to do better than most calculus students. Give yourself the time and space to really dig in.