# Lecture 1

# Logistics and speed

## Logistics

1. We meet every Monday, Wednesday, and Friday at 9 AM. We meet in Bruce and Gloria Ingram Hall 03102.

2. You have a "lab" every Tuesday and Thursday, 9:30 am - 10:50 am, in Bruce and Gloria Ingram Hall 03102.

3. My name is Hiro. My office hours are from 10 AM to 11 AM on Mondays and Wednesdays. My office is MCS 452.

4. Your GA (Graduate Assistant) is Farah Najdawi.

### 1.0.1 No need to buy a textbook

If you need a reference, use the free online textbook of Guichard. A link is on the course website.

### 1.0.2 Grades

Here is the grading rubric:

- Class quizzes: 20 percent
- Lab: 10 percent
- Writing assignments/Homework: 20 percent
- Exam I: 15 percent
- Exam II: 15 percent
- Final exam: 20 percent

### 1.0.3 Quizzes

Almost every lecture, you will be given a quiz at the beginning of class. There may also be quizzes during lab.

### 1.0.4 Exams

There will be three exams in this class.

Exam I: Derivatives, limits, and applications

Exam II: Integrals and applications

Final Exam

### 1.0.5 Resources

There are many resources available to you for the calculus sequence. Make sure to take advantage of them—resources like these are usually not available later in your college careers.

**Online documents**. The math department has posted PDF files of useful facts for people taking Calculus II: `https://www.math.txstate.edu/resources-student/mathcats/course/2472.html`

**Math CATS and CalCentral (Free tutors)**. The math department has calculus tutors ready to help you at Math CATS every day. For more details, see their website: `https://www.math.txstate.edu/resources-student/mathcats`

Finally, you should not be afraid to use Google to find other calculus textbooks in the world.

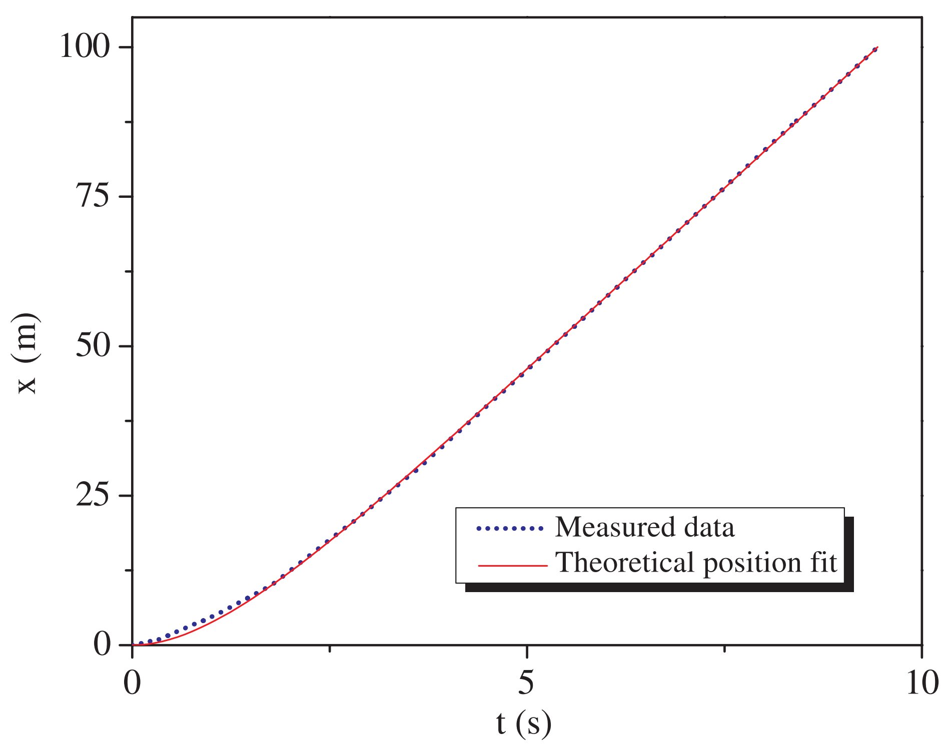
## 1.1 Speed: Usain Bolt

*Source: J J Hernández Gómez et al, 2013 Eur. J. Phys. 34 1227, On the performance of Usain Bolt in the 100 m sprint.*

In 2009 in Berlin (at the World Championships in Athletics), Usain Bolt ran a 100-meter sprint in 9.58 seconds.

(a) What was his overall speed, in meters per second?

(b) Below is a graph depicting Bolt's run. (The dots are the positions at actually measured moments; the solid curve is a curve estimating exact positioning at other times.) Were there times at which Bolt was running *faster* than the speed you computed in part (a)? How do you know?

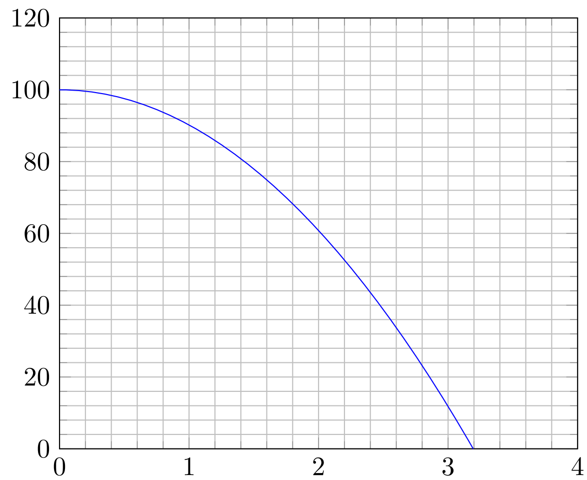



(c) Somebody asks you how fast Bolt was running when he was exactly 5 seconds into the race. How would you estimate this?

Explain carefully.

## 1.2 Speed of a falling ball

Below is a graph depicting the height $f(t)$ of a ball at time $t$.




The horizontal axis is in units of seconds (s), and the vertical axis is measured in meters (m). You can interpret the graph as depicting what happens over time when you drop a ball from 100 meters high.

(a) What is the height of the ball at time $t = 2$ seconds?

(b) Suppose your friend tells you the height of the ball after $2 + h$ seconds. (Here, $h$ is some number.) That is, your friend tells you $f(2 + h)$. In terms of $f(2)$ and $f(2 + h)$, how far does the ball vertically travel between times 2 and $2 + h$?

(c) Over that period of time, what is the average speed of the falling ball?

(d) What does this average speed have to do with the line segment between the point $(2, f(2))$ and the point $(2 + h, f(2 + h))$?

(e) How should you change $h$ if you want to know the speed of the ball *at* time $t = 2$?

# Preparation for Lecture 2

I want you to get practice with expressions that look like the following:

$$\frac{f(x+h)-f(x)}{h}$$

Such expressions are called *difference quotients*. There is a reason these are called difference quotients. First, the numerator measures the *difference* in the value of the function at $x+h$, and the value of the function at $x$. Then, one divides (i.e., takes the *quotient*) of that difference by the distance between $x$ and $x+h$—this is the $h$ in the denominator.

To evaluate difference quotients concretely, you need to know what the function $f$ is.

**Example 1.2.1.** If $f(x) = 2x+1$, then

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} = \frac{(2(x+h)+1)-(2x+1)}{h} &= \frac{2x+2h+1-2x-1}{h} \\ &= \frac{2h}{h} \\ &= 2.\end{aligned}$$

**Example 1.2.2.** If $f(x) = x^2+10$, then

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} = \frac{\big((x+h)^2+10\big)-\big(x^2+10\big)}{h} &= \frac{x^2+2xh+h^2+10-x^2-10}{h} \\ &= \frac{2xh+h^2}{h} \\ &= 2x+h.\end{aligned}$$

**Example 1.2.3.** If $f(x) = x^3$, then

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^3-x^3}{h} &= \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h} \\ &= \frac{3x^2h+3xh^2+h^3}{h} \\ &= 3x^2+3xh+h^2.\end{aligned}$$

Once you know what $f$ is, you should be able to tell me what the difference quotient is as an expression of $x$ and of $h$.

For next class (when you will have another quiz), you should be able to evaluate (and show work for!) the difference quotients of the following functions:

1. $f(x) = 5x+3$
2. $f(x) = x^2+x$
3. $f(x) = x^3+2$
4. $f(x) = x$