Quiz 4 solutions

Let $f(x) = (5x^2 + 7x)/x$. The limit of $g(x)$ as x approaches zero is 7.

Now let $\epsilon = 0.1$. Find a positive number δ so that, if $|x| < \delta$, then $f(x)$ is within ϵ of 7.

We want to know when

$$|f(x) - 7| < \epsilon.$$

Simplifying the lefthand side, we see

$$\begin{aligned} |f(x) - 7| &= |\frac{5x^2 + 7x}{x} - 7| \\ &= |\frac{5x^2 + 7x}{x} - \frac{7x}{x}| \\ &= |\frac{5x^2 + 7x - 7x}{x}| \\ &= |\frac{5x^2}{x}| \\ &= |5x|. \end{aligned}$$

(Technically, we are assuming that $x \neq 0$ to avoid the division-by-zero issue.) So, putting this all together, we want to know when $|5x| < \epsilon$. Diving both sides by 5, we see that this happens precisely when $|x| < \epsilon/5 = (0.1)/5 = 0.02$. In fact, any δ less than $\epsilon/5$ would do.