Quiz 6 solutions

Use the composition law, and the fact that $h(x) = \sqrt{x}$ is continuous, to prove the following case of the root law: If $\lim_{x\to a} f(x)$ exists, then

$$\lim_{x\to a} \sqrt{f(x)} = \sqrt{\lim_{x\to a} f(x)}.$$

In your work, indicate **where** you are using the composition law, and **why the continuity of** $h(x)$ is necessary.

Let $h(x) = \sqrt{x}$. Then

$$\begin{aligned} \lim_{x\to a} \sqrt{f(x)} &= \lim_{x\to a} h(f(x)) \\ &= h(\lim_{x\to a} f(x)) \\ &= \sqrt{\lim_{x\to a} f(x)}. \end{aligned}$$

We use the composition law in the second line (when we move the limit "inside" h). The composition law is only true when h (the outside function) is continuous, so that's why we needed that h is continuous.