Extra Credit Assignment 8

Due Friday, April 9, 11:59 PM

. Fix two topological spaces X and Y, along with a point $x_0 \in X$. Suppose $f : X \to Y$ is a continuous function. We will set $y_0 := f(x_0)$.

In a previous extra credit assignment, you saw that the function

$$f_\sharp : \pi_1(X, x_0) \to \pi_1(Y, y_0), \qquad [\gamma] \mapsto [f \circ \gamma]$$

was a group homomorphism.

(a) Suppose that f is homotopic to g rel x_0. (This means that there exists a continuous function
$$H : [0,1] \times X \to Y$$
for which $H(t, x_0) = y_0$ for every $t \in [0,1]$.) Prove that $f_\sharp = g_\sharp$.

(b) If f is a homotopy equivalence, show that $f_\sharp$ is a bijection.