Extra Credit Assignment 10

Due Friday, April 23, 11:59 PM

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Define $\mathbb{R}^\infty$ to be the direct sum of $\mathbb{R}$ with itself, countably infinitely many times.

(i) Can you put a natural metric on $\mathbb{R}^\infty$?

In class, we said that if $X = \bigcup_n X_n$, one can topologize X so that $U \subset X$ is open iff for all n, $U \cap X_n$ is open in X_n.

(ii) Explain how you can think of $\mathbb{R}^\infty$ as a union of $\mathbb{R}^n$ for $n \geq 0$. Compare the metric topology from (i) to the natural topology of $\mathbb{R}^\infty$ as a union of $\mathbb{R}^n$s.