Commitment to being a good educator

This is a math class. And I will speak, repeatedly, of what it means to be a mathematician. Part of my job this semester is to show you what it is to be a mathematician—in practice, through knowledge, and with confidence.

But let me state the obvious, which is that you are human beings first. And what I care much more about, much more than the math, is that you are able every day to walk upright as human beings.

If anything in your life is preventing you from learning, that is *our* problem; if not passing this class delays your life in anyway, it is our problem; if I am doing anything that prevents you from learning with dignity, that is our problem.

So as I commit to being a good educator for each and every one of you; I want you to commit to tell me if I am not being a good educator. Your commitment will make my commitment a lot less empty.

Lecture 1

Some questions I want you to investigate

Question 1.0.1. How do you know that the surface of the earth is, roughly, a sphere?

Question 1.0.2. At this exact moment, are there two points on the surface of the earth

(a) with exactly the same temperature?

(b) with exactly the same atmospheric pressure?

(c) with exactly the same atmospheric pressure and temperature?

Question 1.0.3. You might think about the surface of the earth in at least two ways: (i) Using a map, or (ii) Visualizing it as a sphere.

Each time you play Pacman, you are looking at a map of Pacman's world. (So we conceptualize Pacman's world using method (i).) Can you conceptualize Pacman's world using method (ii)?

Question 1.0.4. Let $S^3 \subset \mathbb{R}^4$ be the set of all points in $\mathbb{R}^4$ a unit distance from the origin. Explicitly:

$$S^3 = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \,|\, x_1^2 + x_2^2 + x_3^2 + x_4^2 = 1\}.$$

Consider also the space $S^1 \times S^1 \times S^1$. Could these two spaces be homeomorphic? Why or why not? How can you tell?

Question 1.0.5. Let P^1 be the collection of lines in $\mathbb{R}^2$ that pass through the origin. How is P^1 related to the unit circle, if at all?

Let P^2 be the collection of lines in $\mathbb{R}^3$ that pass through the origin. How is P^2 related to the unit circle, if at all?

Question 1.0.6. Consider the set of all real 2-by-2 matrices. Does this set feel like a space homeomorphic to a space you know?

Consider the set of all orthogonal, real, 2-by-2 matrices with determinant one. Is it homeomorphic to a space you know? (A matrix A is called orthogonal if $A^T A$ is the identity matrix.)

Consider the set of all unitary, complex, 2-by-2 matrices with determinant one. Is it homeomorphic to a space you know? (A matrix A is called unitary if $A^* A$ is the identity matrix. Here, A^* is the conjugate transpose of A.)