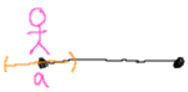
Topology on $\mathbb{R}^4$:

INTUITION: If $A \subset X$ is open, any $x \in A$ has wiggle room.

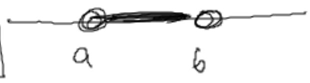
Recall that a topology on a set X is just a declaration of which subsets of X deserve to be called "open."



$[a,b]$

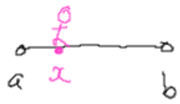
(Standard topology on $\mathbb{R}$)

<u>Ex</u> Some subsets of $\mathbb{R}$ are called "open intervals."



$(a,b) =]a,b[$

More generally, let's call a subset of $\mathbb{R}$ open if it can be expressed as a union of open intervals.



$\forall x \in]a,b[$,
$\exists \varepsilon > 0$ s.t.

$]x-\varepsilon, x+\varepsilon[\; \subset \;]a,b[$.

More generally, if $A \subset \mathbb{R}$ is open, then
$\forall x \in A$
$\exists \varepsilon > 0$ s.t.
$]x-\varepsilon, x+\varepsilon[\; \subset A$.

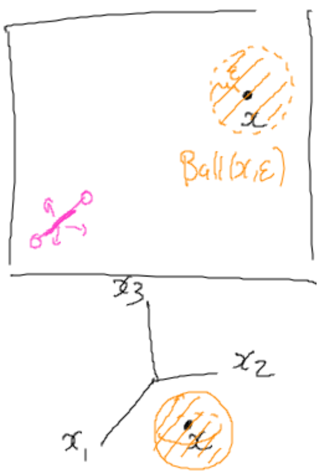
Standard topology on $\mathbb{R}^4$

<u>Defn</u> Fix $x \in \mathbb{R}^4$.

Fix $\varepsilon > 0$.

The <u>open</u> <u>ball</u> of radius ε centered @ x is the set

$$\mathrm{Ball}(x,\varepsilon) = \{ y \in \mathbb{R}^4 \mid \mathrm{dist}(x,y) < \varepsilon \}.$$



We say that $A \subset \mathbb{R}^4$ is <u>open</u> if $\forall x \in A$, $\exists\ \varepsilon > 0$, s.t.

$$\mathrm{Ball}(x,\varepsilon) \subset A.$$

<u>Exer</u> $\Longleftrightarrow$ A is a union of open balls.