$$S^1 \times S^1 \longleftarrow R^2 \longrightarrow S^2$$

$$(e^{i\theta_1}, e^{i\theta_2}) \longleftarrow (\theta_1, \theta_2)$$

$(\cos\theta, \sin\theta)$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\boxed{e^{i(\theta+\theta')}} = e^{i\theta}e^{i\theta'}$$

$$\cos(\theta+\theta') + i\sin(\theta+\theta')$$

$$e^{i\theta}e^{i\theta'}$$

$$= (\cos\theta + i\sin\theta)\cdot(\cos\theta' + i\sin\theta')$$

$$= \cos\theta\cos\theta' + i\sin\theta\cos\theta' + i\sin\theta'\cos\theta + i^2\sin\theta\sin\theta'$$

$$= \cos\theta\cos\theta' + i\sin\theta\cos\theta' + i\sin\theta'\cos\theta - \sin\theta\sin\theta'$$

$$= (\cos\theta\cos\theta' - \sin\theta\sin\theta') + i(\sin\theta\cos\theta' + \sin\theta'\cos\theta)$$



$(-1, 0, 0)$

$\pi$

$\theta_1 = 0$

$(1, 0, 0)$

$$\mathbb{R}^2 \times S^1 \longleftarrow \mathbb{R}^2 \longrightarrow S^2$$

$$\left(r(\theta_1), e^{i\theta_2}\right) \longleftarrow\!\!\!\mapsto \left(\cos\theta_1, \underbrace{\sin\theta_1 \cos\theta_2}, \underbrace{\sin\theta_1 \sin\theta_2}\right)$$

$$\mathbb{R}^2 \times \mathbb{R}^2 = \mathbb{R}^4$$

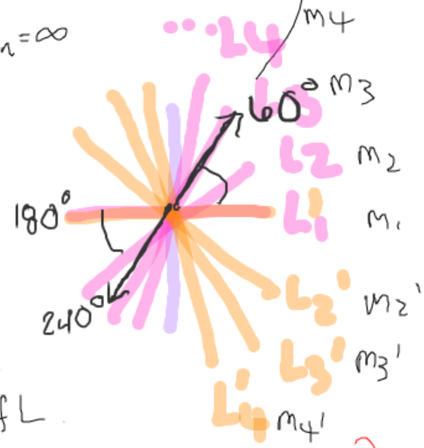

CR

$$P^1 = \{ L \subset \mathbb{R}^2 \mid L \text{ is a line, } (0,0) \in L \}$$

$\lim\limits_{x \to 0^+} \frac{1}{x} = \frac{1}{0^+}$ $\quad \frac{1}{0^-} = \lim\limits_{x \to 0^-} \frac{1}{x}$ $\quad \lim\limits_{n \to \infty} m_n' = -\infty$ $\quad \lim\limits_{n \to \infty} m_n = \infty$








Acevedo: "$P^1 \xrightarrow{\text{slope}} \mathbb{R}$"

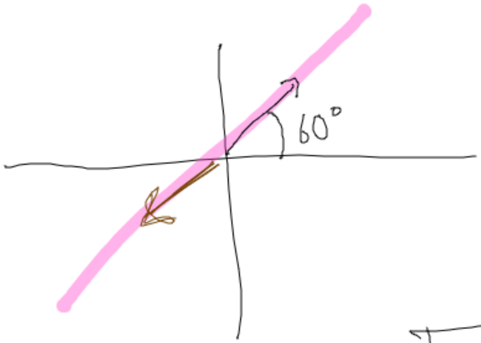
$L \longmapsto$ slope of $L$

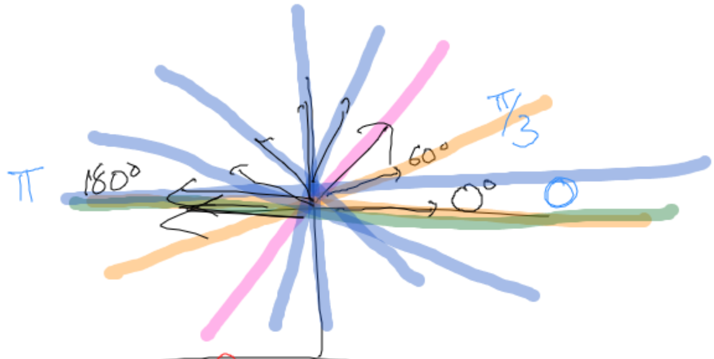
$P^1 \setminus \{\text{the vertical line}\} \longrightarrow \mathbb{R}$

$L \longmapsto$ slope of $L$

① Is $P^1$ the one-point compactification of $\mathbb{R}$?

② and that $\cong$ to $S^1$?

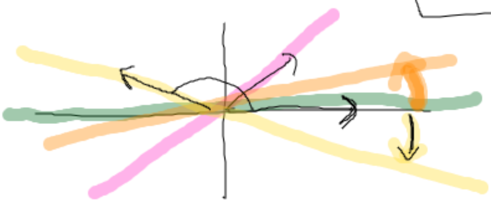


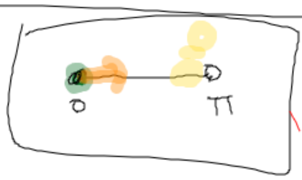



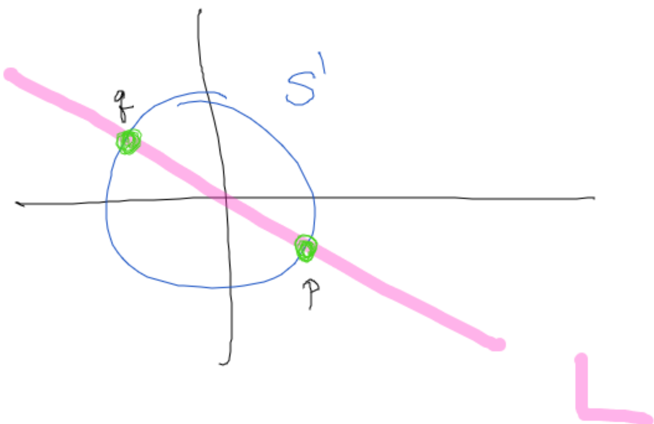

$[0, \pi)$ $\longleftarrow$ $P^1 \longrightarrow [0, 2\pi] / \theta \sim \theta + \pi$

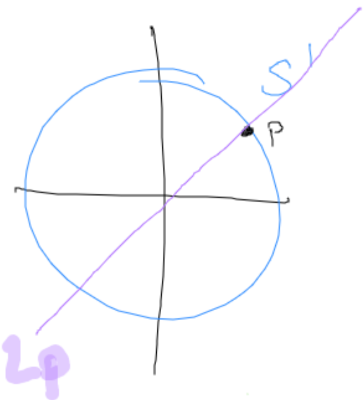
is this a homeom?

$\overset{?}{\cong} S^1$








$L \in \mathbb{P}^1 \rightsquigarrow \{p, q\} \subset S^1$ $\qquad$ $p \in S^1 \rightsquigarrow L_p \in \mathbb{P}^1$