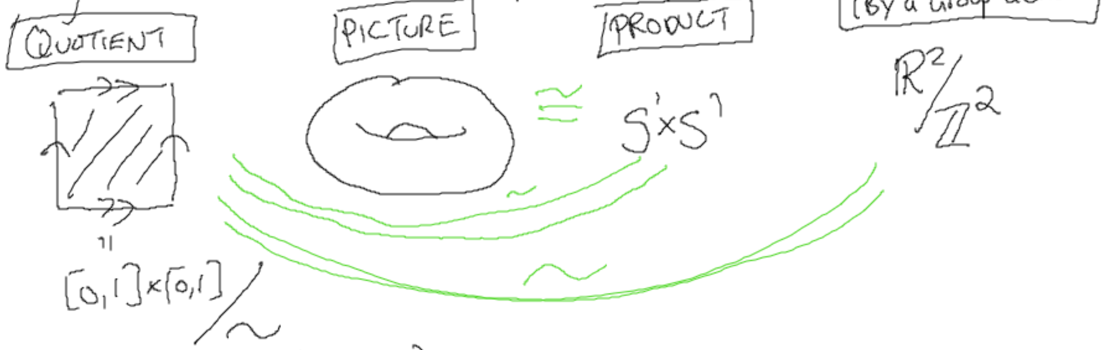
Review:

Motivating Q: Can we tell spaces apart?

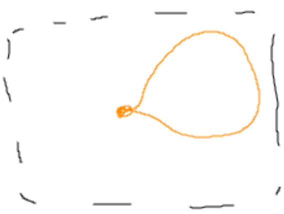


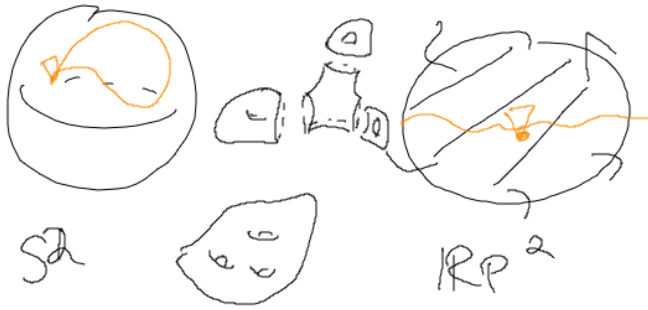
We created an invariant called $\pi_1(X, x_0)$

↳ If X is homeom. to Y, these invariants are isomorphic.

So we could conclude: If $\pi_1(X, x_0)$ is NOT isom. to $\pi_1(Y, y_0)$, then there is NO homeomorphism from X to Y taking x_0 to y_0.

The primary way we can <u>explore</u> what $\pi_1(X, x_0)$ is: drawing pictures

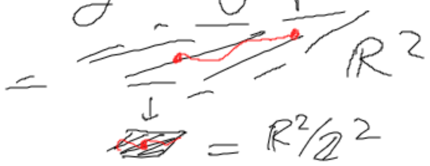


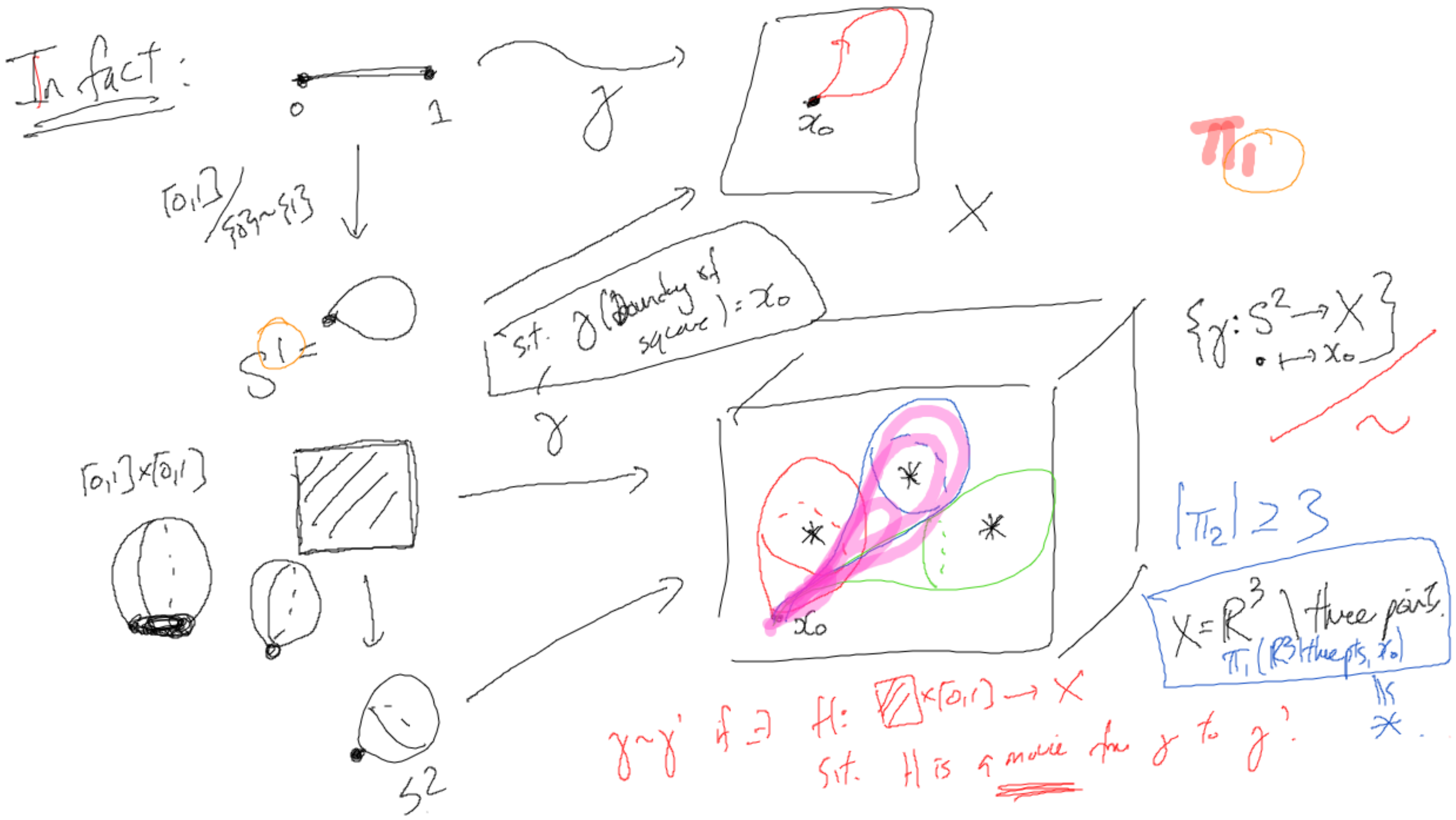


VAN KAMPEN THEOREM

The primary way we can <u>prove</u> what $\pi_1(X, x_0)$ is: By exhibiting X as a quotient by a group action.

(If $\pi_1(\tilde{X}) \cong *$ and $G \times \tilde{X} \to \tilde{X}$ is a nice group action — and if $\tilde{X}$ is nice — then $\pi_1(\tilde{X}/G) \cong G$.)





In fact:
0
1
γ
x₀
X
π₁
[0,1]/{0}~{1}
S¹ =
s.t. γ(boundary of square) = x₀
γ
[0,1]×[0,1]
S²
{γ: S² → X}
• ↦ x₀
/~
|π₂| ≥ 3
X = R³ \ three points
π₁(R³\three pts, x₀)
≅
*
γ~γ' if ∃ H: ▨×[0,1] → X
s.t. H is a movie from γ to γ'.

We say "$\gamma \sim \gamma'$", "γ is htpic to γ' rel x_0". $\pi_2(X, x_0) := \{\gamma\}/\sim$.

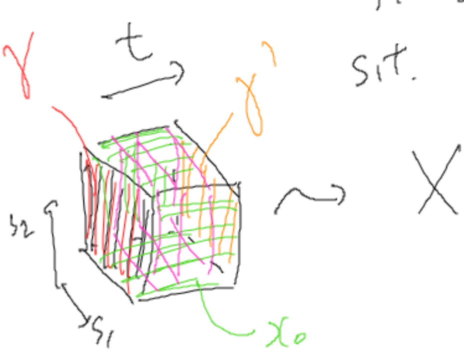
DEF. A homotopy rel x_0 from γ to γ'

$\gamma, \gamma' : [0,1]\times[0,1] \to X$

s.t. $\gamma\left(\begin{smallmatrix}\text{bdy}\\ \text{of sq}\end{smallmatrix}\right) = x_0$

IS: a cont. fxn

$$H: \underbrace{[0,1]\times[0,1]}\times[0,1] \longrightarrow X$$



s.t.
- $H(s_1, s_2, 0) = \gamma(s_1, s_2)$
- $H(s_1, s_2, 1) = \gamma'(s_1, s_2)$
- $H(0, s_2, t) = H(1, s_2, t)$

$\forall s_1, s_2, t.$ $= H(s_1, 0, t)$

$= H(s_1, 1, t) = x_0$

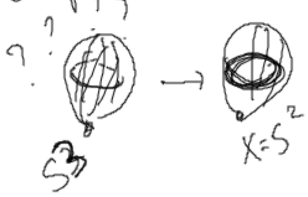
$$\pi_n(X, x_0) := \{ [0,1]^n \xrightarrow{\gamma} X \text{ s.t. } \gamma(\text{bdy of cube}) = x_0 \} \,/\, \text{homotopy rel } x_0$$

Is abelian if $n \geq 2$.

WIDE OPEN PROBLEM

Given $n, m \geq 0$,

compute $\pi_n(S^m, x_0)$



$\pi_2(S^2, x_0) \cong \mathbb{Z} \ni k$

$\pi_3(S^2, x_0) \cong \mathbb{Z}$ Hopf

$\pi_n(S^n) \cong \mathbb{Z}$

$\pi_n(S^m)$

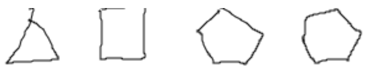
is π_{n+1} bigger in size than π_n?

$\pi_{n+1}(S^m)$

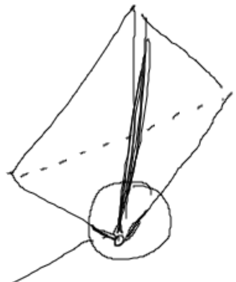
0 0 ∞ 0 C_{24} $C_{24} \times C_2$ 0 0 ∞

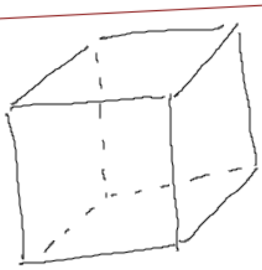
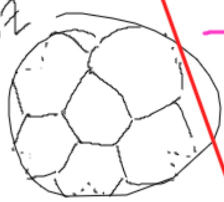
(Regular)

Platonic Solids



A Polyhedron whose
- faces are all congruent
- corners are all congruent
- edges are all congruent



	V-E+F	V	E	F
Tetrahedron ArM 	2 2 2 2	4	6	4
xnZ 	2 2	8 8 8	12 12 12	6 6 6
ArM OCTAHEDRON 	~~2~~ 2	6	12	8
DODECAHEDRON xnZ 	2 2 2	~~22?~~ :(20 20	~~32?~~ ? 30 30	12 12 ✓ 12 12

$3 \times V = \#\text{Arrows} = 60$

$\Rightarrow V = 20$

$2 \times E = \#\text{Arrows} = 5 \times 12$

$= 60$

$\Rightarrow E = 30$