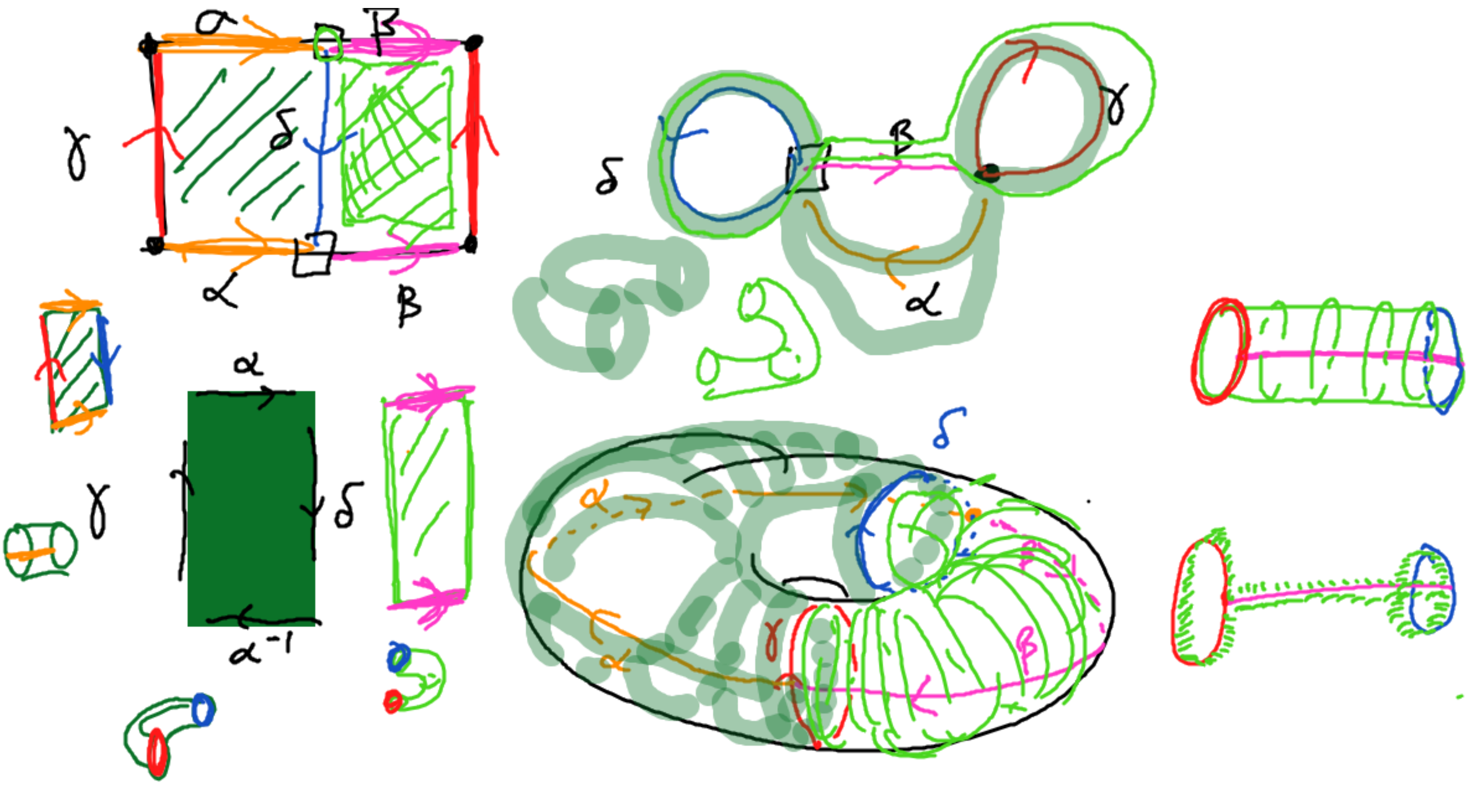
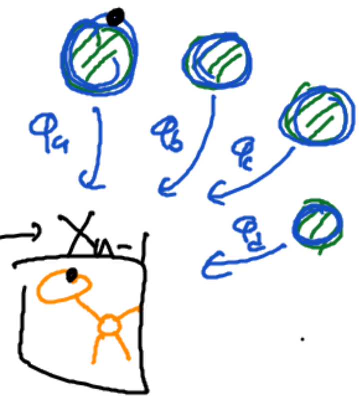

α
β
γ
δ
α
β
α
δ
γ
α⁻¹
δ
β
γ
δ
α
δ
α
α
γ
β

A CW cplx X is

$$X_0 \subset X_1 \subset X_2 \subset \cdots$$

$$X = X_0 \cup X_1 \cup X_2 \cup X_3 \cup \cdots = \bigcup_{n \geq 0} X_n$$

$\boxed{n=2}$



To make X_n, we choose

- A_n = cllxn of n-cells
- $\forall\ a \in A_n$, $\varphi_a : \partial D^n \to X_{n-1}$

and set

given QUOTIENT TOP.

$$X_n = X_{n-1} \cup \left(\bigcup_{A_n} D^n \right) \Big/ \ \forall\, x \in \partial D^n_a,\ x \sim \varphi_a(x)$$

(The · union topology
· direct limit topology)

DEFN If $X = \bigcup_{n \geq 0} X_n$ and each X_n is a top. space,

We say $U \subset X$ is open iff $\forall n$, $U \cap X_n$ is open.

Last time:

set of n-cells

Given a CW cplx X $\left(A_0, A_1, A_2, A_3, \cdots, A_n, \cdots \right.$
$\forall\, a \in A_n$, a cts fxn
$\left. \varphi_a : \partial D^n \to X_{n-1} \right)$

made abelian gps

- $C_n(X) := (\mathbb{Z}/2\mathbb{Z})^{\oplus A_n}$ $\Big(\cong \underbrace{\mathbb{Z}/2\mathbb{Z} \oplus \cdots\cdots \oplus \mathbb{Z}/2\mathbb{Z}}_{\# A_n} \Big)$

$(1,1) = (1,0)+(0,1)$

$\cup\!\!\!\!|$ $(0,0,\cdots,0,1,0,\cdots,0)$

- CLAIM ONE: We have ab gp maps
$$\partial_n : C_n(X) \to C_{n-1}(X)$$
$$(0,\cdots,0,1,0,\cdots,0) \longmapsto ???$$

- CLAIM TWO: $\partial_{n-1} \circ \partial_n = 0$.

DEFN

$$H_n(X; \mathbb{Z}/2\mathbb{Z}) := \operatorname{Ker}(\partial_n) / \operatorname{Im}(\partial_{n+1})$$

Ex 0-dimensional CW cplxs.

A_0

$X_0 = A_0 =$ 

$\emptyset = A_1 = A_2 = A_3 = A_4 = \cdots$

if A_0 is finite

$$\longrightarrow C_0 = (\mathbb{Z}/2\mathbb{Z})^{\oplus A_0} = \overbrace{\mathbb{Z}/2\mathbb{Z} \oplus \cdots \oplus \mathbb{Z}/2\mathbb{Z}}^{\#A_0}$$

$(\mathbb{Z}/2\mathbb{Z})^{\oplus 7}$ 2^7

$$C_1 = (\mathbb{Z}/2\mathbb{Z})^{\oplus A_1} = (\mathbb{Z}/2\mathbb{Z})^{\oplus \emptyset} = 0.$$

$2^0 = 1$

$\boxed{H_0 = ?}$

$$C_n = (\mathbb{Z}/2\mathbb{Z})^{\oplus A_n} = 0.$$

$$C_0 \xleftarrow{\partial_1} C_1 \xleftarrow{\partial_2} \underset{C_2}{\overset{0}{\parallel}} \xleftarrow{\boxed{\partial_3}} \underset{C_3}{\overset{0}{\parallel}} \xleftarrow{\partial_4} \underset{C_4}{\overset{0}{\parallel}}$$

the triv. gp.

$$\boxed{\forall n \geq 1,\ H_n(X; \mathbb{Z}/2\mathbb{Z}) \cong 0.}$$

$$H_3(X; \mathbb{Z}/2\mathbb{Z}) := \operatorname{Ker}(\boxed{\partial_3}) \big/ \operatorname{Im}(\partial_4) \cong 0/0 \cong 0.$$

$$H_0(X;\mathbb{Z}/2\mathbb{Z}) := \mathrm{Ker}(\partial_0)/\mathrm{Im}(\partial_1) \cong C_0/0$$

$$\cong C_0$$

$$\cong (\mathbb{Z}/2\mathbb{Z})^{\oplus A_0}$$

<u>DEFN</u> For any CW cplx, ∂_0 is the trivial map.

$$0 \xleftarrow{\partial_0} \boxed{C_0} \xleftarrow{\partial_1} \overset{0}{\overset{\shortparallel}{C_1}}$$

$$\boxed{\dim H_0(X;\mathbb{Z}/2\mathbb{Z}) = \#A_0 \text{ when } A_0 \text{ finite}}$$

<u>UPSHOT</u>

If $X = X_0$, H_n is stupid. $H_1 = H_2 = H_3 = \cdots = 0$ and H_0 is some $(\mathbb{Z}/2\mathbb{Z})^{\oplus \# pts}$.

$H_*(S^1; \mathbb{Z}/2\mathbb{Z})$. (ie, compute all $H_n(S^1; \mathbb{Z}/2\mathbb{Z})$)

$$\Big(H_* := \bigoplus_{n \geq 0} H_n \Big).$$

$X_0 = \bullet$ $\quad A_0 = \{a\}$ $\quad \varphi_e : \partial D^1 \longrightarrow X_0$

$\| $

$X_1 =$  $\quad A_1 = \{e\}$

$C_n := (\mathbb{Z}/2\mathbb{Z})^{\oplus A_n}$

$$\boxed{\begin{array}{l} C_0 = \mathbb{Z}/2\mathbb{Z} \\ \uparrow \partial_1 \\ C_1 = \mathbb{Z}/2\mathbb{Z} \end{array}}$$

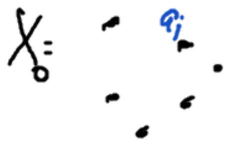
To make ∂_1, fix an element $e \in A_1$. Consider the basis vector

$$(0, 0, \ldots, 0, \underset{\substack{\uparrow \\ e^{th} \text{ spot.}}}{1}, 0, \ldots, 0) \in C_1.$$

$$\partial_1\Big(\underbrace{(0,\dots,0,1,0,\dots,0)}_{e^{\text{th}}\text{ spot}}\Big) = \big(\varepsilon_{a_1}, \varepsilon_{a_2}, \varepsilon_{a_3}, \varepsilon_{a_4}, \dots\big)_{a_i\in A_0}$$

$e \in A_1$

$a_i \in A_0$



$$\partial_1 : C_1 \to C_0 = \bigoplus_{A_0} (\mathbb{Z}/2\mathbb{Z})$$

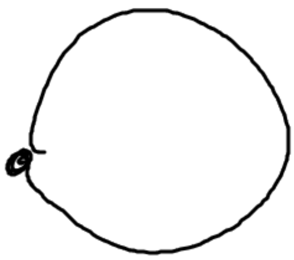
where ε_{a_i} is the # of times φ_e hits a_i.

$\boxed{\text{DEFN of } \partial_1}$

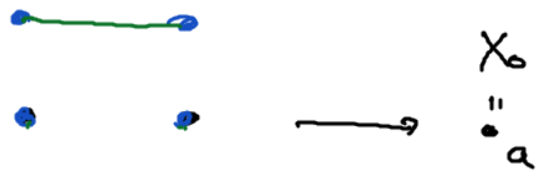
$(\varepsilon_{a_1}, \varepsilon_{a_2}, \dots, \cdot)$

0 or 1

$a_i \in A_0$.



φ_e:



$$\varphi_e(1) = 2 = 0 \in C_0 = \mathbb{Z}/2\mathbb{Z}$$

$$1 \in \mathbb{Z}/2\mathbb{Z} = C_1$$

$$C_0 \xleftarrow{\partial_1} C_1 \leftarrow C_2 \leftarrow \cdots$$

$$0 \leftarrow \mathbb{Z}/2\mathbb{Z} \xleftarrow{0} \mathbb{Z}/2\mathbb{Z} \leftarrow 0 \leftarrow \cdots$$