Writing Assignment 3

Due Monday, February 8, 11:59 PM

For last week's extra credit, you had to look up the definition of topology. Let me remind you that a topology on a set X is a collection $\mathcal{T}$ of subsets of X. This collection must satisfy the following properties: (i) The empty set and X are elements of $\mathcal{T}$, (ii) Any union of elements of $\mathcal{T}$ is again an element of $\mathcal{T}$, and (iii) Any *finite* intersection of elements of $\mathcal{T}$ is an element of $\mathcal{T}$.

Consider the following two metrics on $\mathbb{R}^2$:

$d_{std}(x, x') = \sqrt{(x_1 - x_1')^2 + (x_2 - x_2')^2}$

$d_{taxi}(x, x') = |(x_1 - x_1')| + |x_2 - x_2'|$.

Show that a subset of $\mathbb{R}^2$ is open with respect to d_{std} (the standard metric) if and only if it is open with respect to d_{taxi} (the taxicab metric).

(This could, of course, inform how we think about the different metrics on $M_{2\times 2}(\mathbb{R})$, and whether the affect the topology of $M_{2\times 2}(\mathbb{R})$.)