2. Is there something wrong?

Background:

- A number is called *positive* if it is strictly greater than 0. Put another way, x is positive exactly when $x > 0$.
- A positive integer is called a *prime number* if its only divisors are one and itself. In other words, if the only other positive integers that can divide the number—without any remainder—are 1 and the number itself.
- Finally, an integer is called *odd* if it is not divisible by 2. (In base 10, this is equivalent to the assertion that the 1's digit is equal to 1, 3, 5, 7, or 9.[1])

Below are claims one can find on the internet, and the reasoning behind each claim (also that one might find on the internet). As you read them, be critical, and think about whether the claims are convincing to you.

(a) **Claim: Every odd number is a prime number.**
Reasoning: 3 is an odd number and it's prime. 5 is an odd number and it's prime. 7 is an odd number and it's prime. See? It took me less than fifteen seconds to convince you.

(b) **Claim: Every prime number is an odd number.**
Reasoning: 3 is a prime number and it's odd, 5 is a prime number and it's odd, 7 is a prime number and it's odd. See? It's almost too easy!

(c) **Claim: There is no such thing as a largest integer.**
Reasoning: Suppose there were a largest integer. Add one to it to get a new number. This new number is bigger! So anybody who tries to tell you that there is a largest number has to be wrong. See?

(d) **Claim: There is no such thing as a largest real number.**
Reasoning: Suppose there were a largest real number. Let's call it x. Well, there's another real number called $x + 1$. We obviously have the inequality $x + 1 > x$, so the so-called "largest" real number x wasn't really the largest. So anybody who tries to tell you that there is a largest real number has to be wrong. See?

(e) **Claim: There is no such thing as a largest rational number.**
Reasoning: Suppose there were a largest rational number. Let's call it x. Well, there's another rational number called $x + 1$. We obviously have the inequality $x + 1 > x$, so the so-called "largest" rational number x wasn't really the largest. So anybody who tries

[1] You should spend some time thinking about why!

to tell you that there is a largest rational number has to be wrong. See?

(f) **Claim: There is no such thing as a smallest, positive, rational number.**

Reasoning: Suppose there were a smallest, positive, rational number. Call it a*. Then* $a/2$ *is another rational number, and we have that* $0 < \frac{a}{2} < a$*. In other words,* $a/2$ *is smaller than* a*, and it's stil positive! So anybody who claims there is a smallest, positive rational number has to be wrong.*

(g) **Claim: There is no such thing as a smallest, positive, real number.**

Reasoning: Suppose there were a smallest, positive, real number. Call it a*. Then* $\frac{1}{a}$ *is a real number, and so is* $B = \frac{1}{a} + 1$*. Note that* $B > \frac{1}{a}$*. Therefore, we conclude that* $\frac{1}{B} < a$*. But look!* $\frac{1}{B}$ *is a positive real number, and it's smaller than* a*! See?*

(h) **Claim: There is no such thing as a smallest, positive integer.**

Reasoning: Suppose there were a smallest positive integer. Call it a*. Then* $\frac{1}{a}$ *is an integer and so is* $B = \frac{1}{a} + 1$*. Note that* $B > \frac{1}{a}$*. Therefore, we conclude that* $\frac{1}{B} < a$*. But look!* $\frac{1}{B}$ *is a positive integer, and it's smaller than* a*! So anybody who tries to tell you that there is a smallest positive integer has to be wrong.*

Prompt. For each of the eight claims above, answer the following: Is there something wrong about the reasoning used to justify the claim? If so, what? If not, why not?

For your homework submission, write a complete response to the above prompt, giving full reasoning. If you do not have an answer, you must explain what explorations you undertook—and you must write at least three appreciably different things that you tried to explore potential answers.